Test ID: TESTID Name: _____

MAT 300 **Quiz 2** Spring 2016

1. Find the derivative: $\frac{d}{dt}B_5^7(t)$.

   a) $7(B_4^6(t) - B_5^6(t))$ b) $8(B_3^6(t) + B_4^6(t))$ c) $7(B_4^6(t) - B_3^6(t))$ d) $7(B_3^6(t) - B_4^6(t))$ e) $8(B_4^6(t) - B_3^6(t))$

   Correct Answer: $7(B_4^6(t) - B_5^6(t))$

2. Choose a correct equivalent expression for $B_2^5(t)$:

   a) $\frac{d}{dt}\frac{1}{6}C_2^6(t)$ b) $B_1^4(t) + B_2^4(t)$ c) $(1-t)B_3^5(t) + tB_4^5(t)$ d) $tB_1^4(t) + (1-t)B_2^4(t)$ e) $\frac{d}{dt}6(B_3^6(t) - B_4^6(t))$

   Correct Answer: $tB_1^4(t) + (1-t)B_2^4(t)$

3. If a Newton form $p(t) = 2 + 3(t-1) - 4(t-1)(t-2)$ matches a data function $g(t)$ at $t = 1$, $t = 2$, and $t = 3$, what is $[1, 2, 3]g$?

   a) 3 b) -3 c) -4 d) 4 e) 2

   Correct Answer: -4

4. Same $p(t)$ and $g(t)$ as in the previous question. What is $[1, 2]g$?

   a) 3 b) -3 c) 3 d) 2 e) 4

   Correct Answer: 3

5. Use a simple observation (about the graph) to find the interpolating polynomial in P_2 that passes through the points $(-1, 2)$, $(3, 0)$, and $(5, -1)$. In the standard basis, with $p(t) = a_0 + a_1t + a_2t^2$, what is the coefficient a_1?

   a) -1 b) $-\frac{1}{2}$ c) $\frac{1}{2}$ d) 1 e) 0

   Correct Answer: $-\frac{1}{2}$

6. To show the existence and uniqueness of the interpolating polynomial $p(t) = a_0 + a_1t + a_2t^2 + \cdots a_dt^d$, for a data sequence $t_0, \ldots, t_d$, and data function $g(t)$, with y-values $y_i = g(t_i)$, using the standard basis, we used a linear system $A\mathbf{x} = \mathbf{b}$. The appropriate entries of the column vector $\mathbf{x}$ are:

   a) $a_0, \ldots, a_d$ b) $t_0, \ldots, t_d$ c) $y_0, \ldots, y_d$ d) $y_0 - t_0, \ldots, y_d - t_d$ e) $\frac{a_1-a_0}{t_1-t_0}, \ldots, \frac{a_d-a_{d-1}}{t_d-t_{d-1}}$

   Correct Answer: $a_0, \ldots, a_d$

7. Same $A\mathbf{x} = \mathbf{b}$ as in the previous question. The existence and uniqueness of $p(t)$ follows from:

   a) $det(A) \neq 0$ b) $det(A) = 0$ c) $g(t_i) = 0$, $i = 0, \ldots, d$ d) $g(t_i) \neq 0$, $i = 0, \ldots, d$ e) $t_i \neq 0$, $i = 0, \ldots, d$

   Correct Answer: $det(A) \neq 0$

8. Below is an interpolating polynomial written with Lagrange polynomials, that passes through the points: $(1, 2)$, $(2, 4)$, $(4, 1)$. Find the correct value of the missing constant C:

$$(2)\frac{(t-2)(t-4)}{(1-2)(1-4)} + (4)\frac{(t-1)(t-4)}{(2-1)(2-4)} + (1)\frac{(t-1)(t-2)}{C}$$

   a) -2 b) -6 c) 2 d) 6 e) 3

   Correct Answer: 6

9. The polynomial $p(x) = \begin{vmatrix} 1 & x & x^2 \\ 1 & 3 & 9 \\ 1 & 5 & 25 \end{vmatrix}$ has zeros at x equal to:

   a) 3 and 2 b) 1 and 3 c) 1 and 5 d) 3 and 9 e) 3 and 5

   Correct Answer: 3 and 5

10. Same $p(x)$ as in the previous question. The leading coefficient of $p(x)$ is:

   a) 4 b) 5 c) 1 d) 2 e) 3

   Correct Answer: 2