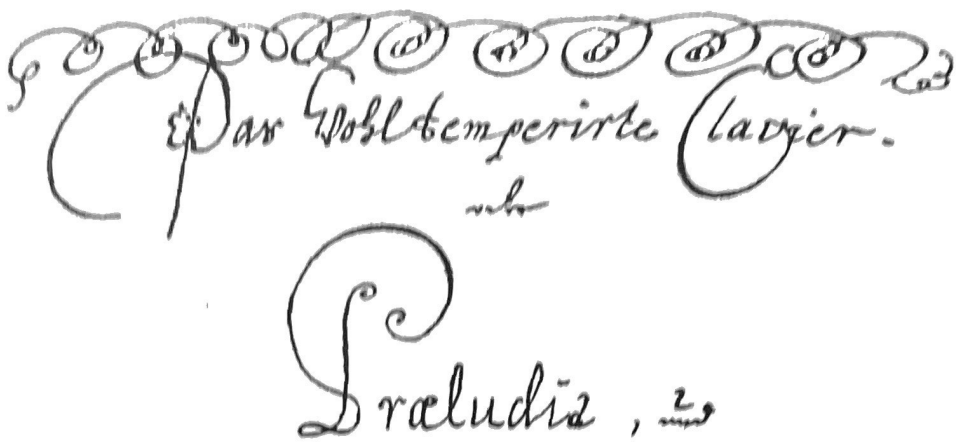

1 The top of the title-page of Bach's *Das wohltemperirte Clavier* (1722). The complete page is reproduced in the 1911 *Grove dictionary*, vol. i, p. 152, and in many modern editions of the music. (Staatsbibliothek zu Berlin Preußischer Kulturbesitz)

(then or now!) setting up different selections on different occasions? What other instruments and voices are to be used with the keyboard? What is the role of musical taste and experience? Are the instruments to be left in a particular temperament for years, needing only minor maintenance, or to be retuned afresh on every occasion? To understand the nature of these problems of keyboard temperament along with Bach's solution, a brief review of the basic tuning issues and nomenclature is required.

All octaves are pure, as a rule. Within that premise, the tuning of 12-note keyboards presents three competing problems. The tuner must find a musically appropriate and tasteful distribution of three different error intervals. These intervals are as follows:

Pythagorean comma (hereafter 'PC'): twelve pure 5ths, less seven octaves; the interval $^{531441}/_{524288}$. A cycle of twelve consecutive 5ths must *remove* a total of one PC, to avoid overshooting the octave. Since there are twelve 5ths, some or all of them must be tuned too narrow by some portion(s) of the PC. On average the 5ths must be 1/12 PC too flat.

Syntonic comma (hereafter 'SC'): four pure 5ths, less two octaves and a pure major 3rd; the interval $^{81}/_{80}$. A cycle of four consecutive 5ths must *remove* one SC if the resulting major 3rd *is* to be pure. Since there are twelve major 3rds, some or all of them must be tuned too wide by some portion(s) of the SC. On average the major 3rds must be 7/11 SC too sharp. (This is a large and easily audible amount: approximately 1/7 of an equal-tempered semitone.)

Lesser diesis (hereafter simply 'diesis'): an octave, less three pure major 3rds; the interval $^{128}/_{125}$. For example: if we tune three pure 3rds C–E, E–G♯, G♯–B♯ we get the interval C–B♯ which is much too flat to be an octave. As octaves must be pure, the diesis must be distributed to one or more of the three major 3rds making some or all of them wider than pure. This must be handled four times in any temperament: in the major 3rd stacks starting on F, C, G and D.

The geometric difference between the PC and the SC is called the *schisma*. It is very nearly the same size as either 1/12 PC or 1/11 SC.

Temperaments usually split either the PC or the SC into small, manageable portions of 1/12, 1/6, 1/3 or 1/4. These are distributed among some or all of the 5ths. The 5ths are made deliberately out of tune by these subtle amounts, such that the resulting 3rds and steps will have a pleasing balance in their own musical functions. For all practical purposes the 1/12 PC temperaments are 1/11 SC temperaments, as those two small portions have very nearly the same size.

Mathematical modelling of the same: scientific reproducibility

To get the temperament *exactly* the same from day to day, and for mathematical analysis, it can be set onto the keyboard in a slightly different sequence. As with a problem-solving technique of mazes (working both forward and backward simultaneously), a tuner can set a temperament onto the keyboard in whatever sequence helps to get the job done accurately and comfortably, as long as the resulting layout is correct. Try the following step-by-step instructions:

1 Establish the naturals F–C–G–D–A–E in regular ⅙ PC; that is, set up the first half of 'Vallotti', which is already familiar to harpsichordists.[27] The most practical single thing to know is that the major 3rd of F–A (both in the tenor octave below middle C) has about 3 beats per second.[28] A second checkpoint is middle C up to E, with about 4.5 beats per second.[29]

2 Pure 5ths: E–B–F♯–C♯.

3 Pure 5ths: F–B♭–E♭. (This is a step to put E♭/D♯ exactly where it belongs.)

4 Put G♯/A♭ in place against C♯ and E♭, slightly tempered from each (i.e. 1/12): the single *Spuren* in Bach's diagram.

5 Go back to B♭ and lower it slightly, so that D♯–A♯ is similarly a 1/12 5th as shown by Bach's diagram, and B♭–F becomes a slightly wide 5th.

6 All octaves and unisons are pure.

The results are shown in table 1; compare again with illus.1 and 2 to see the correspondence, and play through all the 5ths in the sequence of Bach's drawing. The lesson is learned.

For those who are most comfortable setting temperaments from an electronic device, the bottom two rows of the chart show the adjustments from equal temperament, to the nearest cent, reckoned either from C or A. The deviations from regular ⅙ Pythagorean comma and regular ⅙ syntonic comma are also given, for keyboardists and ensembles accustomed to those standards.

In the results from Bach's drawing we have a set of the usual and regular ⅙ comma naturals; a pure B obtained from E; and then the remaining five accidentals are each tastefully adjusted: sharps raised slightly to build a remarkably high G♯, and flats lowered slightly from their classic positions . . . giving complete flexibility across the keyboard. The aural magic happens in Bach's specific details of those tasteful nudges. Since my April 2004 discovery of this method, it has already received a thorough workout and confirmation beyond my own regular use. Andrew Manze has toured with it for orchestral continuo,[30] and George Taylor has built it into the new Goshen College organ.[31]

Why hasn't this been found before?

The following digest from the standard tuning literature and *WTC* literature should make it obvious why Bach's diagram has not been treated as evidence for any particular unequal temperament. Historiography, while allowing for the possibility that Bach had a specific practical temperament, has pointed away from investigation of unequal temperaments, and away from any evidence that is outside the realm of words.

Is Bach's drawing on the *WTC* title-page worth any more than a quick glance? The *Neue Bach-Ausgabe* (*NBA*) officially dismissed the spiral diagram, three times, as merely an ornamental flourish. (1) *Bach-Dokumente*, 1963: 'Titelseite: Wohltemperiertes Klavier, Köthen, 1722. (. . .) Autograph. (. . .) [Ü]ber dem Text hs. Ornamente, unter dem Text großer Schlußschnörkel . . .'[32] (2) *Kritischer Bericht* (*KB*), 1989: 'Titel, siehe Notenband, S. XIV und 1. (. . .) Über und unter dem Titel ornamentale Schnörkel.'[33] The *KB*, as always with the *NBA*, is so thorough and imposing with detail, representing so many years of close scrutiny by the world's top Bach scholars, it is surprising that the single most important piece of information at the top of the title-page could have been missed. (3) The *Notenband* itself (also 1989) presents a facsimile of the title-page, of course. But, on the opposite page of the same spread, only the words of Bach's macaronic introducion (and their layout) are reproduced in modern transcription.[34] The reader's attention is drawn immediately to the legible modern transcription of the words, and away from the photograph.

The Bach reader (1945; rev. edn 1966) presents the title-page in a clear English translation, preserving