MAT 120 — Homework 2 — Fall 2024

Due date: Thursday, Sep 26

Note: An equal tempered major third is four semitones. A just major third has frequency ratio 5/4.

1. Find the frequency ratio, and the cent value for that frequency ratio, for the following pairs of notes (to two decimal places):
   (a) Note 1: two octaves plus an equal tempered major third above $A440$ and Note 2: four equal tempered perfect fifths above $A440$
   (b) Note 1: two octaves plus a just major third above $A440$ and Note 2: four just perfect fifths above $A440$
   (c) E_1 and E_2, where E_1 is the open E string on a violin tuned with three equal tempered fifths above $G3$, and E_2 is the same E string tuned with three just perfect fifths above $G3$.
   (d) E_1 and E_2, where E_1 is the open high E string on a guitar tuned with an equal tempered major third plus an equal tempered fourth above $G3$, and E_2 is the same E string tuned with a just major third plus a just perfect fourth above $G3$.
2. Find the Just Frequency Ratios, and their corresponding cent values, for all the remaining intervals implied in the Just Major Scale. For instance, we already saw all of the ratios starting from the first note, and we already know the steps between consecutive notes. Now find the values for the steps between more than two notes in different parts of the scale. (For example: start with the second note and find the interval from there to the third note, then the fourth, up to the octave. Then start on the third note, etc.)
3. How many Equal-Tempered semitones fit into the human audible frequency range? (Assume this is 20 Hz to 20 kHz.) Equivalently: How many (Equal-Tempered) keys would a piano need in order to span the human audible frequency range?

The cent formula for frequency ratio F_2/F_1:

$$\textit{cent value} \quad x = \frac{1200}{\ln 2} \ln\left(\frac{F_2}{F_1}\right)$$

This is equivalent to the exponential form:

$$2^{\frac{x}{1200}} = \frac{F_2}{F_1}$$

The Just Diatonic Scale

$$\frac{1}{1} \longrightarrow \frac{9}{8} \longrightarrow \frac{5}{4} \longrightarrow \frac{4}{3} \longrightarrow \frac{3}{2} \longrightarrow \frac{5}{3} \longrightarrow \frac{15}{8} \longrightarrow \frac{2}{1}$$

$$\left(\frac{9}{8}\right)\ \left(\frac{10}{9}\right)\ \left(\frac{16}{15}\right)\ \left(\frac{9}{8}\right)\ \left(\frac{10}{9}\right)\ \left(\frac{9}{8}\right)\ \left(\frac{16}{15}\right).$$