MAT 120 — Homework 5 — Fall 2024

Due date: Thursday, Nov.7

1. For two harmonic tones with the given fundamental frequencies F_1 and F_2, find the first example of harmonics which will beat at some number less than 20 beats per second. Say which harmonic number for each of the tones is the one that beats with the other harmonic from the other tone. (Note: call the fundamental the first harmonic.) If there is no such beat pattern within the first 10 harmonics of each tone, then say NO BEAT PATTERN. Give the number of beats to one decimal place accuracy. For the following problems, use A with fundamental frequency 220 Hz, and use C for middle C. Assume all intervals below are Equal Tempered.
   (a) $F_1 = A$, F_2 is a whole tone above F_1.
   (b) $F_1 = A$, F_2 is a minor third above F_1.
   (c) $F_1 = A$, F_2 is a major third above F_1.
   (d) $F_1 = A$, F_2 is a perfect fourth above F_1.
   (e) $F_1 = A$, F_2 is a perfect fifth above F_1.
   (f) $F_1 = A$, F_2 is a major sixth above F_1.
   (g) $F_1 = C$, F_2 is a whole tone above F_1.
   (h) $F_1 = C$, F_2 is a minor third above F_1.
   (i) $F_1 = C$, F_2 is a major third above F_1.
   (j) $F_1 = C$, F_2 is a perfect fourth above F_1.
   (k) $F_1 = C$, F_2 is a perfect fifth above F_1.
   (l) $F_1 = C$, F_2 is a major sixth above F_1.

2. Write down five different types of major third, saying where they come from or what name they have, and also their frequency ratio and their cent value. For each major third, determine how many beats per second are induced if you play a harmonic tone at 110 Hz and a major third above it. (The major thirds should come from the Just, Equal-Tempered, Pythagorean, or Mean-Tone scales, at least one from each, and can occur between any two scale degrees which are four semitones apart.)

3. Equal Temperament can be summarized as "12 Equal Divisions of the Octave". This means that the smallest step is a semitone and there are twelve of them per octave. Find the cent value of the smallest step in each of the following divions of intervals:
   (a) 10 equal divisions of the octave
   (b) 4 equal divisions of a Just Major Whole Tone
   (c) 16 equal divisions of a Just Perfect Fifth
   (d) 8 equal divisions of a syntonic comma

4. Find the following special values of sinusoids:

   (a) $\cos(\frac{11\pi}{4})$
   (b) $\sin(\frac{11\pi}{4})$
   (c) $\cos(-\frac{17\pi}{6})$
   (d) $\sin(-\frac{17\pi}{6})$
   (e) $\cos(\frac{7\pi}{3})$
   (f) $\sin(\frac{7\pi}{3})$

5. Find the values using a trig identity and special values:

   (a) $\sin^2(\frac{\pi}{4}) + \cos^2(\frac{\pi}{4})$
   (b) $\sin^2(\frac{\pi}{4}) + \cos^2(-\frac{\pi}{4})$
   (c) $\sin(\frac{\pi}{4} + \frac{\pi}{6})$
   (d) $\cos(\frac{\pi}{4} - \frac{\pi}{6})$
   (e) $\sin(\frac{\pi}{12})$

6. For each problem rewrite the product as a sum of trig functions.

   (a) $\sin(2\pi t)\cos(\frac{3}{2}\pi t)$
   (b) $\sin(2\pi t)\sin(2\pi t)$
   (c) $\sin(3\pi t)\sin(8\pi t)$

7. For each problem rewrite the sum as a product of trig functions. Then find the frequency of each of the factors, and say which factor is an audible frequency and which factor is a low frequency oscillator. Also state how many beats per second this sum (or product) will have.

   (a) $\sin(200\pi t) + \sin(210\pi t)$
   (b) $\cos(330\pi t) + \cos(338\pi t)$