

MAT 120

Midterm Exam

Fall 2020

1. Which of the following are rational numbers?

- i) .010110111 ii) $2^{1/2} \cdot 2^{1/2}$ iii) $(\frac{3}{2})^7$
 a) all of them b) i) and ii) only c) iii) only d) i) and iii) only e) i) only

Correct Answer: all of them

2. Which of the following numbers are complex non-real? Here i is the square root of -1 . (Hint: simplify first, where possible.)

- i) $\sqrt{1} - \sqrt{2}$ ii) $(1+i)(1-i)$ iii) $\sqrt{-2}^3$
 a) all of them b) i) and ii) only c) i) only d) ii) only e) iii) only

Correct Answer: iii) only

3. The number $1 + \frac{33}{100} \left(1 + \frac{1}{100} + \frac{1}{10000} + \dots \right)$ can be simplified to which of the following?

- a) $\frac{4}{3}$ b) 2 c) $\frac{13}{10}$ d) 3 e) $\frac{33}{10}$

Correct Answer: $\frac{4}{3}$

4. Suppose that $0 < r < 1$. Simplify $1 + (1-r)(1+r+r^2+r^3+\dots)$:

- a) 1 b) $1+r$ c) 2 d) $\frac{1}{1-r}$ e) r

Correct Answer: 2

5. The number $2\sqrt{2}$ represents a frequency ratio for an interval. How many *whole* tones is this interval?

- a) 5 b) 9 c) 7 d) 14 e) 36

Correct Answer: 9

6. Suppose we attempt to create a major scale with only: 1) the Just Major whole tone $\frac{9}{8}$, and 2) the Just semitone $\frac{16}{15}$, and usual pattern of whole and half steps. How far off will we be at the octave? Express your answer to the closest half cent.

- a) 21.5 b) 10 c) 11.5 d) 54 e) 43

Correct Answer: 43

7. A frequency ratio of 10 corresponds to a just interval of two octaves and two perfect fifths plus which interval?

- a) minor whole tone b) minor third c) major third d) major whole tone e) perfect fourth

Correct Answer: minor whole tone

8. A frequency ratio of 10 is about how many semitones?

- a) 37 b) 40 c) 41 d) 39 e) 38

Correct Answer: 40

9. Approximately how many equal-tempered semitones fit into the audible range of a fox, assuming this goes from 20 Hz to 200000 Hz?

- a) 160 b) 300 c) 120 d) 420 e) 440

Correct Answer: 160

10. If F_1 and F_2 are frequencies, with $F_1 < F_2$, and the frequency ratio F_2/F_1 is measured as x cents, then these numbers can be related in which of the following formulas:

a) $x \frac{2}{1200} = \frac{F_2}{F_1}$ b) $1200 \cdot 2^x = \frac{F_2}{F_1}$ c) $x = 1200 \cdot \ln \left(\frac{F_2}{F_1} \right)$ d) $1200 \cdot \ln 2 = \frac{F_2}{F_1}$ e) $2 \frac{x}{1200} = \frac{F_2}{F_1}$

Correct Answer: $2 \frac{x}{1200} = \frac{F_2}{F_1}$

11. Approximately how many just semitones with frequency ratio $\frac{25}{24}$ are there in one Just Perfect Fifth?

a) 8 b) 7 c) 10 d) 11 e) 9

Correct Answer: 10

12. Suppose a guitar is tuned so that the low E string is at E2 and the high E string is at E4, two octaves higher. Suppose that the B string is tuned a just perfect fourth lower than the high E string, and the A, D, and G strings are tuned by just perfect fourths consecutively up from the low E string. How far off from an Equal Tempered Major third will the interval from the G to the B string be, as an approximate value in cents?

a) 8 b) 16 c) 21.5 d) 3.5 e) 2

Correct Answer: 8

13. In the meantone scale, with starting frequency 220 Hz, let F_1 be the frequency which is closest to $\frac{4}{3}220$ and let F_2 be the frequency which is closest to $\frac{5}{3}220$. What is the frequency ratio F_2/F_1 ?

a) $\frac{5}{4}$ b) $\sqrt{\frac{5}{3}}$ c) $\sqrt{2}$ d) $\frac{5}{4}\sqrt{2}$ e) $\sqrt{\frac{5}{4}}$

Correct Answer: $\frac{5}{4}$

14. Suppose a microtone is defined to be an equal division of the just major sixth into sixteen equal parts. Approximately how many such microtones make up one equal tempered tritone?

a) 6 b) 9 c) 8 d) 11 e) 12

Correct Answer: 11

15. A harmonic tone X is played together with another harmonic tone Y such that Y has fundamental frequency which is 1.3 times that of X . If X has fundamental frequency 100 Hz, how many beats should we hear per second if we are listening to the first 5 harmonics of each tone but no higher?

a) 8 b) 6 c) 10 d) 4 e) 2

Correct Answer: 10

16. In tuning a keyboard using the circle of fifths, suppose we repeat with a perfect fifth which has value 696 cents, by starting on middle C and going up by fifths, then copying the new notes onto all octaves. How far off, in cents, from an equal tempered whole tone will the interval from G to A be?

a) 8 b) 10 c) 6 d) 4 e) 2

Correct Answer: 8

The Meantone Major (Diatonic) Scale

$$\frac{1}{1} \rightarrow \sqrt{\frac{5}{4}} \rightarrow \frac{5}{4} \rightarrow \frac{\sqrt{2}}{\left(\frac{5}{4}\right)^{\frac{1}{4}}} \rightarrow \sqrt{2} \left(\frac{5}{4}\right)^{\frac{1}{4}} \rightarrow \sqrt{2} \left(\frac{5}{4}\right)^{\frac{3}{4}} \rightarrow \sqrt{2} \left(\frac{5}{4}\right)^{\frac{5}{4}} \rightarrow \frac{2}{1}$$

$$\left(\sqrt{\frac{5}{4}}\right) \quad \left(\sqrt{\frac{5}{4}}\right) \quad \left(\frac{\sqrt{2}}{\left(\frac{5}{4}\right)^{\frac{1}{4}}}\right) \quad \left(\sqrt{\frac{5}{4}}\right) \quad \left(\sqrt{\frac{5}{4}}\right) \quad \left(\sqrt{\frac{5}{4}}\right) \quad \left(\frac{\sqrt{2}}{\left(\frac{5}{4}\right)^{\frac{1}{4}}}\right)$$

The Just (Natural) Chromatic Scale

$$\frac{1}{1} \rightarrow \frac{16}{15} \rightarrow \frac{9}{8} \rightarrow \frac{6}{5} \rightarrow \frac{5}{4} \rightarrow \frac{4}{3} \rightarrow \frac{64}{45} \rightarrow \frac{3}{2} \rightarrow \frac{8}{5} \rightarrow \frac{5}{3} \rightarrow \frac{16}{9} \rightarrow \frac{15}{8} \rightarrow \frac{2}{1}$$

$$\left(\frac{16}{15}\right) \left(\frac{135}{128}\right) \left(\frac{16}{15}\right) \left(\frac{25}{24}\right) \left(\frac{16}{15}\right) \left(\frac{16}{15}\right) \left(\frac{135}{128}\right) \left(\frac{16}{15}\right) \left(\frac{25}{24}\right) \left(\frac{16}{15}\right) \left(\frac{135}{128}\right) \left(\frac{16}{15}\right)$$

The Pythagorean Diatonic Scale

$$\frac{1}{1} \rightarrow \frac{9}{8} \rightarrow \frac{81}{64} \rightarrow \frac{4}{3} \rightarrow \frac{3}{2} \rightarrow \frac{27}{16} \rightarrow \frac{243}{128} \rightarrow \frac{2}{1}$$

$$\left(\frac{9}{8}\right) \left(\frac{9}{8}\right) \left(\frac{256}{243}\right) \left(\frac{9}{8}\right) \left(\frac{9}{8}\right) \left(\frac{9}{8}\right) \left(\frac{256}{243}\right)$$