MAT 250   **Quiz 3**   Spring 2026

1. Let $\mathbf{u} = (1, -2, 3)^T$ and $\mathbf{v} = (2, 1, 1)^T$ be two (column) vectors in $\mathbb{R}^\mathbf{3}$. Choose another vector $\mathbf{w}$ so that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ make a linearly independent set. Hint: if $\mathbf{w} = (a, b, c)^T$ then applying Gaussian Elimination to the matrix of column vectors $A = [\mathbf{u}\ \mathbf{v}\ \mathbf{w}]$ produces:

$$\begin{pmatrix} 1 & 2 & a \\ -2 & 1 & b \\ 3 & 1 & c \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & a \\ 0 & 1 & \frac{1}{5}(2a + b) \\ 0 & 0 & a - b - c \end{pmatrix}$$

a) $(3, 2, 1)^T$ b) $(1, 1, 0)^T$ c) $(0, 1, -1)^T$ d) $(3, 0, 3)^T$ e) $(2, 0, 1)^T$

Correct Answer: $(2, 0, 1)^T$

2. Same information as in the previous question but now choose $\mathbf{w}$ so that the linear system $A\mathbf{x} = \mathbf{0}$ has only the zero solution $\mathbf{x} = \mathbf{0}$.

a) $(2, 2, 1)^T$ b) $(1, 1, 0)^T$ c) $(0, 1, -1)^T$ d) $(3, 0, 3)^T$ e) $(1, 0, 1)^T$

Correct Answer: $(2, 2, 1)^T$

3. Same information as in the previous two questions but now choose $\mathbf{w}$ so that the rows of A are linearly dependent.

a) $(3, 2, 1)^T$ b) $(1, 2, 0)^T$ c) $(0, 1, -2)^T$ d) $(2, 0, 3)^T$ e) $(1, 2, 1)^T$

Correct Answer: $(3, 2, 1)^T$

4. Let S be a set of 3 vectors in $\mathbb{R}^3$. Which of the following statements must be true:

i) S is linearly independent

ii) S spans $\mathbb{R}^3$

iii) S is a basis of $\mathbb{R}^3$

a) i) only   b) ii) only   c) i) and ii) only   d) ii) and iii) only   e) none of the above

Correct Answer: none of the above

5. Let S be a set of 3 linearly *dependent* vectors in $\mathbb{R}^3$. Which of the following statements must be true:

i) some vector in S is a linear combination of other vectors in S

ii) some vector in S is a scalar multiple of another vector in S

iii) S does not span $\mathbb{R}^3$

a) i) only   b) ii) only   c) i) and ii) only   d) ii) and iii) only   e) i) and iii) only

Correct Answer: i) and iii) only