# MAT 250 Homework 2
# Spring 2026

Due date: Thursday, January 22

Note: you may use a symbolic math package such as PARI or SAGE, or a calculator that does linear algebra, to work on these problems. But the work that you turn in should show all your your steps. Be sure to check ALL your work. This is simple to do with such a package. Wrong answers will receive zero.

1. Solve the linear system in two ways: a) reducing the matrix to row-echelon form and using back substitution, and b) finding the reduced row-echelon form and writing the answer.

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$

2. Find the reduced row-echelon form of the augmented matrix of this system and then write the general solution in column vector format.

$$\begin{array}{rrrrrrcr} x_1 & +3x_2 & -2x_3 & & +2x_5 & & = & 0 \\ 2x_1 & +6x_2 & -5x_3 & -2x_4 & +4x_5 & -3x_6 & = & -1 \\ & & 5x_3 & +10x_4 & & +15x_6 & = & 5 \\ 2x_1 & +6x_2 & & +8x_4 & +4x_5 & +18x_6 & = & 6 \end{array}$$

3. Find a linear system of 2 equations in 3 variables that has general solution $x = 1$, $y = 2$, $z = t$ for any real number $t$. Write the augmented matrix of the system in reduced row-echelon form.

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5. Find a linear system of 2 equations in 3 variables that has no solutions. Write the augmented matrix of the system in reduced row-echelon form.

6. Show that a linear system of 2 equations in 2 variables, with coefficient matrix $A$ cannot have exactly one (one and only one) solution if $det(A) = 0$. Label all the coefficients in the augmented matrix and show what can happen in the Gaussian Elimination procedure. Give a criterion for the two possible cases: i) no solution, or ii) infinitely many solutions, in terms of the coefficients, and using $det(A) = 0$.

7. Find a linear system of 3 equations in 2 variables that has general solution $x = 1$, $y = 1$, and such that none of the equations has all coefficients equal to zero. Write the augmented matrix of the system in reduced row-echelon form.

8. Find a linear system of 3 equations in 2 variables that has no solutions, and such that none of the equations has all coefficients equal to zero. Write the augmented matrix of the system in reduced row-echelon form.

9. Use Gaussian Elimination to find the inverse of the matrix: $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$

10. Use Gaussian Elimination to show that the matrix is not invertible: $\begin{pmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{pmatrix}$