MAT 250 Homework 3
Spring 2026

Due date: Friday, January 30

Note: you may use a symbolic math package such as PARI or SAGE, or a calculator that does linear algebra, to work on these problems. But the work that you turn in should show all your your steps. Be sure to check ALL your work. This is simple to do with such a package. Wrong answers will receive zero.

1. For which values of a does the solution of the following system have i) exactly one solution, ii) infinitely many solutions, or iii) no solution?

$$\begin{aligned} x + y + z &= 4 \\ z &= 2 \\ (a^2 - 4)z &= a - 2 \end{aligned}$$

2. For the following augmented matrix, determine which values of a and b produce a system with: i) exactly one solution, ii) a one-parameter family of solutions, iii) a two-parameter family of solutions, or iv) no solutions.

$$\begin{pmatrix} a & 0 & b & | & 2 \\ a & a & 4 & | & 4 \\ 0 & a & 2 & | & b \end{pmatrix}$$

3. Find a matrix K such that $AKB = C$ given the following:

$$A = \begin{pmatrix} 1 & 4 \\ -2 & 3 \\ 1 & -2 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \text{ and }, C = \begin{pmatrix} 8 & 6 & -6 \\ 6 & -1 & 1 \\ -4 & 0 & 0 \end{pmatrix}.$$

4. Solve for the matrix A if $AB = C$, given that:

$$B = \begin{pmatrix} -1 & 0 & 1 \\ 1 & 1 & 0 \\ 3 & 1 & -1 \end{pmatrix} \text{ and }, C = \begin{pmatrix} 1 & 2 & 0 \\ -3 & 1 & 5 \end{pmatrix}.$$

5. Find a cubic polynomial in standard basis form ($y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$) which passes through the points $(0, 3)$, $(1, 3)$, $(-1, 7)$, and $(2, 1)$. Use a 4×4 linear system with the coefficients of the cubic as the variables, and solve. Be sure to check your answer by computing the y-values from the x-values as inputs to the cubic.

6. Find a cubic polynomial in standard basis form ($y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$) which passes through $(1, -2)$ with slope -2 at this point, and through $(-1, 2)$ with slope 2 at this point. Use a linear system with the polynomial and its derivative.