MAT 250 Homework 7
Spring 2026

Due date: Wednesday, March 25

1. Let $\mathbf{u} = (1,1)^T$ and let $\mathbf{v} = (2,-1)^T$. Find two orthonormal bases of $\mathbb{R}^2$, one of which a) contains a scalar multiple of $\mathbf{u}$ and the other which b) contains a scalar multiple of $\mathbf{v}$. In each case, use only the vectors $\mathbf{u}$ and $\mathbf{v}$ and operations of projection, linear combination, and normalization.

2. For each subset S of $\mathbb{R}^2$ determine if the set is a subspace or not and if it is not, state which property it fails and why. (Note: when a set is given as points (x,y), you can consider those to be endpoints of vectors.)

    (a) S is the unit circle, the set of points (x,y) with $x^2 + y^2 = 1$.
    (b) S is the set of points (x,y) on any one of the four parabolas: $y = x^2$, $y = -x^2$, $x = y^2$, $x = -y^2$.
    (c) S is the set of points on the line $y = x + 1$.
    (d) S is the set of points on the line $y = x$.
    (e) S is the set of points on the line $y = x$ or $y = -x$.
    (f) S is the set of points (x,y) which are on any parabola $y = cx^2$ for any constant c. (Note: so $(1,1)$ is in S since it is on $y = x^2$ and also $(1,3)$ is in S since it is on $y = 3x^2$, etc.)

3. Let $\mathbf{u} = (1,1,1)^T$ and $\mathbf{v} = (1,2,1)^T$ and Let $\mathbf{w} = (1,1,2)^T$. Find an orthonormal basis of $\mathbb{R}^3$ using the Gram-Schmidt procedure, starting with $\mathbf{u}$ normalized, and using only projection, linear combination, and normalization. (Verify your final basis by checking for orthogonality.)

4. Show that complex numbers w and z are linearly dependent (as real vectors) if and only if $w\bar{z} \in \mathbb{R}$. (Note: the linear dependence statement uses only the vector space properties of $\mathbb{C}$, but the criterion in this case uses the multiplication of $\mathbb{C}$. To show the if and only if statement is true, you need to prove two implications. For example: "A if and only if B" means the same as both "A implies B" and "B implies A". So, first assume A and show that B is true, then also assume B and show that A is true.)

5. Find the projection matrix P which gives the orthogonal projection in $\mathbb{R}^2$ onto the line $2x + 3y = 0$. Then compute the determinant of P, and also find the matrices P^2 and P^3.

6. Find the projection matrix A which gives the orthogonal projection in $\mathbb{R}^3$ onto the plane $2x + 3y + 4z = 0$. Then find all points in $\mathbb{R}^3$ which are the same distance as $(1,1,1)$ from this plane and also the same distance from the origin as $(1,1,1)$. Show that these points form a linearly dependent set in $\mathbb{R}^3$. Compute the orthogonal projections of each of these points on the plane. Show that the resulting set of points in also linearly dependent.