

MAT 250 Homework 8

Spring 2026

Due date Monday, April 13

1. Let A be a 6×6 matrix with characteristic equation

$$p(\lambda) = \det(A - \lambda I) = \lambda^2(\lambda - 1)(\lambda - 2)^3.$$

What are the possible dimensions for eigenspaces of A ?

2. Let $A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix}$.

- (a) Find the eigenvalues of A .
- (b) For each eigenvalue λ find the rank of the matrix $A - \lambda I$.
- (c) Is A diagonalizable? Justify your answer.

3. Let $A = \begin{pmatrix} 2 & -3 \\ 1 & -1 \end{pmatrix}$. Determine if A is diagonalizable or not. Show your work.

4. Let $A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix}$. Determine if A is diagonalizable or not. Show your work.

5. Let $A = \begin{pmatrix} 1 & 0 \\ 6 & -1 \end{pmatrix}$. Find the matrix P which diagonalizes A , and find the product $P^{-1}AP$.

6. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. Find the matrix P which diagonalizes A , and find the product $P^{-1}AP$.

7. Let $A = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix}$. Determine if A is diagonalizable, and say why or not why not. If so, then find the matrix P which diagonalizes A , and find the product $P^{-1}AP$.

8. Let $A = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$. Find the matrix P which diagonalizes A , and find the product $P^{-1}AP$, then use this to compute A^{10} .

9. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Show that each of the following are True. Give reasons and show your work.

- (a) If $(a - d)^2 + 4bc > 0$ then A is diagonalizable.
- (b) If $(a - d)^2 + 4bc < 0$ then A is not diagonalizable.