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MAT 250 **Midterm Exam** Spring 2026

1. Find a linear equation in the variables x and y that has general solution: $x = 5 - 2t$, $y = t$.

   a) $2x + 3y = 5$ b) $2x + y = 5$ c) $3x + y = 5$ d) $x + 2y = 5$ e) $4x + 3y = 5$

   Correct Answer: $x + 2y = 5$

2. Find the quadratic equation $y = a_0 + a_1x + a_2x^2$ so that the parabola passes through the points $(1, 4)$, $(-1, 6)$, and $(2, 9)$. (Use a linear system in three variables.) What is a_1?

   a) 0 b) 1 c) -1 d) 2 e) -2

   Correct Answer: -1

3. How many free variables (parameters) does the system with the following augmented matrix have?

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 17 & -1 \\ -1 & 0 & -2 & -5 & -2 \\ 2 & 0 & 4 & 10 & 4 \end{array}\right)$$

   a) 0 b) 1 c) 2 d) 3 e) 4

   Correct Answer: 2

4. For the next three questions, determine if the linear system of equations with the given augmented matrix is consistent or inconsistent, and how many solutions the system has. Then determine which of the statements below are true. Choose the correct list of TRUE statements from the answers.

   i) the system is consistent

   ii) the system is inconsistent

   iii) the system has exactly one solution

   iv) the system has infinitely many solutions

   v) the system has no solution

$$\left(\begin{array}{cc|c} 3 & 5 & 3 \\ 5 & 3 & 5 \end{array}\right)$$

   a) ii),v) b) i),iii) c) i),iv) d) i),v) e) ii),iii)

   Correct Answer: i),iii)

5. (Same instructions as previous question.)

$$\left(\begin{array}{cc|c} 3 & 5 & 7 \\ 6 & 10 & 14 \end{array}\right)$$

   a) i),v) b) i),iii) c) i),iv) d) ii),v) e) ii),iii)

   Correct Answer: i),iv)

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6. (Same instructions as previous question.)

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 3 & -1 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & -1 & -3 & 6 \end{array}\right)$$

a) ii),v) b) i),iii) c) i),iv) d) iii),v) e) ii),iii)

Correct Answer: ii),v)

7. (Same instructions as previous question.)

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 3 & -1 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & -1 & -3 & 7 \end{array}\right)$$

a) ii),v) b) i),iii) c) i),iv) d) iii),v) e) ii),iii)

Correct Answer: i),iv)

8. Find the determinant of the matrix:

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

a) 0 b) 1 c) -2 d) 2 e) -1

Correct Answer: 1

9. Choose a matrix A which satisfies: $A^2 = -I$.

a) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ b) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$ d) $\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$ e) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Correct Answer: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

10. Express the first two row operations on the following system as a single matrix. Hint: the matrix is the product of two elementary matrices, each of which has the effect of producing one zero in the first column.

$$\begin{pmatrix} x + 2y + 3z &= 4 \\ -2x + y + 3z &= 1 \\ 3x + 4y + z &= 0 \end{pmatrix}$$

a) $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ -3 & 0 & 1 \end{pmatrix}$ e) $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}$

Correct Answer: $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}$

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11. Which of the following are NOT equivalent to $(A+B)^2$ for all 2×2 matrices A and B ?

i) $(B+A)^2$ ii) $A^2+2AB+B^2$ iii) $A(A+B)+B(A+B)$

iv) $(A+B)(B+A)$ v) $A^2+AB+BA+B^2$

a) i) and ii) b) ii) only c) ii) and iii) d) iv) and v) e) v) only

Correct Answer: ii) only

12. Let A be an $n \times n$ matrix. Which of the following are equivalent to A being invertible?

i) $det(A) \neq 0$

ii) $A(\mathbf{x}) = \mathbf{0}$ has at least one solution

iii) $rref(A)$ equals the identity matrix

iv) the system $A(\mathbf{x}) = \mathbf{b}$ is consistent for at least one $\mathbf{b}$

v) the rank of A is n.

a) i) and ii) b) i) iii) and v) c) ii) and iii) d) i) iv) and v) e) all of them

Correct Answer: i) iii) and v)

13. Which of the following statements are true for all 3×3 *invertible* matrices A?

i) $A\mathbf{x} = \mathbf{0}$ has exactly one solution: $\mathbf{x} = \mathbf{0}$

ii) $A\mathbf{x} = \mathbf{b}$ has at least one solution $\mathbf{x}$ for any $\mathbf{b}$

iii) $A\mathbf{x} = \mathbf{b}$ has at exactly one solution $\mathbf{x}$ for any $\mathbf{b}$

a) ii) only b) i),iii) only c) i),ii) only d) i) only e) i),ii),iii)

Correct Answer: i),ii),iii)

14. Let A be an $n \times n$ matrix and suppose that $a_{i,j} = 0$ for $i < j$, and let D be the product $a_{1,1} \cdot a_{2,2} \cdots a_{n,n}$. Then which of the following must be True?

a) $\det(A) = D$ b) $D = 0$ c) $\det(A) = 0$ d) $\det(A) \neq 0$ e) $D < 0$

Correct Answer: $\det(A) = D$

15. Let A and B be $n \times n$ matrices and suppose that $C = AB$. Also suppose that the product of row times column $A_{(i)}B^{(j)} = 0$ for all $i > j$. Then C must be:

a) zero matrix b) symmetric c) identity matrix d) upper triangular e) none of these

Correct Answer: upper triangular

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16. Let A and B be $n \times n$ matrices and suppose that $b_{i,j} = a_{i,j} + a_{j,i}$ for all $i \neq j$. Then it must be true that:

a) $det(B) = 0$ b) B is invertible c) $det(B) > 0$ d) B is symmetric e) none of these

Correct Answer: B is symmetric

17. Let $\mathbf{u} = (1, 1, 0)^T$ and $\mathbf{v} = (2, 1, -1)^T$ be two (column) vectors in $\mathbb{R}^3$. Choose another vector $\mathbf{w}$ so that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ make a linearly *dependent* set. Hint: if $\mathbf{w} = (a, b, c)^T$ then applying Gaussian Elimination to the matrix of column vectors $A = [\mathbf{u}\ \mathbf{v}\ \mathbf{w}]$ produces:

$$\begin{pmatrix} 1 & 2 & a \\ 1 & 1 & b \\ 0 & -1 & c \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & a \\ 0 & 1 & a-b \\ 0 & 0 & b-a-c \end{pmatrix}$$

a) $(3, 2, 1)^T$ b) $(0, 1, 1)^T$ c) $(0, 1, -1)^T$ d) $(2, 0, 3)^T$ e) $(2, 0, -1)^T$

Correct Answer: $(0, 1, 1)^T$

18. Same information as in the previous question but now choose $\mathbf{w}$ so that the linear system $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b}$.

a) $(1, 1, 1)^T$ b) $(1, 1, 0)^T$ c) $(2, 1, -1)^T$ d) $(3, 2, -1)^T$ e) $(1, 3, 2)^T$

Correct Answer: $(1, 1, 1)^T$

19. Let S be a set of 3 linearly *dependent* vectors in $\mathbb{R}^3$. Which of the following statements must be true:

i) some vector in S is a scalar multiple of another vector in S

ii) some vector in S is a linear combination of other vectors in S

iii) S does not span $\mathbb{R}^3$

a) i) only b) ii) only c) i) and ii) only d) i) and iii) only e) ii) and iii) only

Correct Answer: ii) and iii) only

20. Let $p(x)$ be the polynomial function defined as the determinant of the matrix

$$\begin{pmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}$$

What are the zeros (or roots) of the polynomial $p(x)$?

a) 2, 3 b) 1, 2 c) 2, 4 d) 3, 4 e) 3, 9

Correct Answer: 2, 3