

## MAT 258

## Quiz 3

June 2021

Note: The words *at least*  $n$  can be interpreted as  $\geq n$ , and the words *at most*  $n$  can be interpreted as  $\leq n$ .

1. How many bit strings of length 7 contain both of the substrings 0111 and 1011 ?

- a) 15                      b) 14                      c) 13                      d) 16                      e) 17

Correct Answer: 13

2. A task consists of lining up 6 people from left to right and then placing a penny, a nickel, a dime, and a quarter into the left or right pocket of 4 of these 6 people. In the end there will be exactly one coin in exactly one pocket of exactly 4 of the 6 people, who are standing in order from left to right. In how many different ways can this task be performed?

- a)  $(6!)^2 4!$                       b)  $(6!)^2 2^3$                       c)  $(6!4!)^2$                       d)  $(6!2^4)^2$                       e)  $(6!)^2 2^4$

Correct Answer:  $(6!)^2 2^3$

3. How many strings of length 10 are there, whose characters are chosen from the digits 0,1, and 2, if there are exactly three 0's ?

- a)  $\binom{10}{3} 2^7$                       b)  $\binom{10}{4} 2^6$                       c)  $\binom{11}{4} 2^7$                       d)  $\binom{10}{3} 2^6$                       e)  $\binom{10}{3} 2^8$

Correct Answer:  $\binom{10}{3} 2^7$

4. How many different strings of length 8 can be made from the letters in the word HALFLIFE ?

- a)  $\frac{8!}{2!2!}$                       b)  $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$                       c)  $2 \cdot 7!$                       d)  $8 \cdot 7 \cdot 6 \cdot 5$                       e)  $\frac{8!}{4!}$

Correct Answer:  $\frac{8!}{2!2!}$

5. How many different strings of length 9 can be made from the letters in the word STARCRAFT ?

- a)  $\frac{9!}{2!2!}$                       b)  $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$                       c)  $9 \cdot 7 \cdot 6$                       d)  $\frac{9!}{3!}$                       e)  $9 \cdot 7!$

Correct Answer:  $9 \cdot 7!$

6. How many solutions does the equation  $x_1 + x_2 + x_3 + x_4 = 8$  have with integers  $x_i \geq 0$ ?

- a)  $\binom{10}{4}$                       b)  $\binom{11}{3}$                       c)  $\binom{11}{4}$                       d)  $\binom{10}{3}$                       e)  $\binom{8}{3}$

Correct Answer:  $\binom{11}{3}$

7. How many solutions does the equation  $x_1 + x_2 + x_3 + x_4 = 8$  have with integers  $x_i \geq 0$  with the additional restrictions:  $x_1 \geq 2$  and  $x_3 \geq 3$ ?

- a)  $\binom{10}{4}$                       b)  $\binom{6}{3}$                       c)  $\binom{6}{4}$                       d)  $\binom{7}{5}$                       e)  $\binom{7}{4}$

Correct Answer:  $\binom{6}{3}$

8. How many solutions does the equation  $x_1 + x_2 + x_3 + x_4 = 7$  have with integers  $x_i \geq 0$  with the additional restrictions:  $x_1 \leq 2$  and  $x_3 \leq 3$ ?

- a) 63                      b) 78                      c) 70                      d) 66                      e) 55

Correct Answer: 66

9. How many different alphabetized (unlabelled) strings using one or two blanks in place of one or two letters can be formed from the word ESTATES? (Two blanks can be used to replace two of the same letter or two different letters.)

- a) 13                      b) 18                      c) 10                      d) 16                      e) 15

Correct Answer: 13

10. How many different draws from a scrabble bag will result in the ability to spell the word ESTATES assuming that none of the tiles is a blank? (Take into account the frequency of each letter, as in the file scrabble-bag.txt on the website.)

- a) 11880                      b) 19440                      c) 71280                      d) 42768                      e) 53640

Correct Answer: 53460 (all answers received credit since there was a typo)

11. If two fair dice are rolled, what is the probability that at least one odd number shows up?

- a)  $\frac{3}{4}$                       b)  $\frac{1}{3}$                       c)  $\frac{5}{9}$                       d)  $\frac{7}{9}$                       e)  $\frac{2}{3}$

Correct Answer:  $\frac{3}{4}$

12. If two fair dice are rolled, what is the probability that at least one odd number shows up, given that at least one of the numbers is at most 4?

- a)  $\frac{3}{4}$                       b)  $\frac{1}{3}$                       c)  $\frac{5}{9}$                       d)  $\frac{7}{9}$                       e)  $\frac{2}{3}$

Correct Answer:  $\frac{3}{4}$