Test ID: TESTID Name: ____________________

MAT 258 **Quiz 4** July 2021

1. Which of the following is a tautology?

   i) $[(p \to q) \wedge p] \to q$ ii) $[(p \to q) \wedge \neg q] \to \neg p$ iii) $[(p \to q) \wedge \neg p] \to \neg q$

   a) i) and ii) only b) i) and iii) only c) ii) and iii) only d) ii) only e) i) only

   Correct Answer: i) and ii) only

2. Suppose a bit string of length two is generated at random, and the values of the bit string are interpreted as T/F values for two propositions p and q, in that order. (So for example "01" would mean that p is False and q is True.) What is the probability that the proposition $p \longrightarrow q$ is True?

   a) $\frac{7}{8}$ b) $\frac{1}{8}$ c) $\frac{1}{4}$ d) $\frac{3}{8}$ e) $\frac{3}{4}$

   Correct Answer: $\frac{3}{4}$

3. Suppose a bit string of length three is generated at random, and the values of the bit string are interpreted as T/F values for three propositions p, q and r, in that order. What is the probability that the proposition $(p \longrightarrow q) \vee \neg r$ is True?

   a) $\frac{3}{4}$ b) $\frac{7}{8}$ c) $\frac{1}{8}$ d) $\frac{1}{4}$ e) $\frac{3}{8}$

   Correct Answer: $\frac{7}{8}$

4. Which of the following is equivalent to: $\neg(\forall x(\exists y : P(x, y)))$?

   a) $\exists y : \forall x(\neg P(x, y))$ b) $\forall x(\exists y : \neg P(x, y))$ c) $\forall x(\forall y(\neg P(x, y))$ d) $\exists x : \forall y(\neg P(x, y))$ e) $\exists x : \exists y : \neg P(x, y))$

   Correct Answer: $\exists x : \forall y(\neg P(x, y))$

5. Which of the following is equivalent to: $\neg(\forall x(\exists y : \forall z(x + y > z)))$?

   a) $\exists x : \forall y(\exists z : x + y \leq z)$ b) $\exists y : \forall x(\forall z(x + y \leq z))$ c) $\forall x(\exists y : (\exists z : x + y > z)$ d) $\forall x(\forall y(\exists z : x + y > z)$ e) $\exists x : \exists y : (\exists z : x + y \leq z)$

   Correct Answer: $\exists x : \forall y(\exists z : x + y \leq z)$

6. Choose the description that best fits the following statement and proof:

   A positive integer which is greater than 3 and is one less than a perfect square, cannot be a prime number. Proof: Let $m = n^2 - 1$, for some integer $n > 2$. Then $m > 7$, and so m must have at least two factors which are both greater than one. So m is not prime.

   a) direct proof b) indirect proof c) incorrect proof d) proof by contradiction e) counterexample

   Correct Answer: incorrect proof

7. Let p be the statement "there are infinitely many prime numbers", and q be the statement "The prime numbers consist of the finite set $S = \{p_1, p_2, \ldots, p_n\}$", and r be the statement "$N = p_1 p_2 \cdots p_n + 1$", and let u be the statement "$N \in S$". In the proof that there are infinitely many prime numbers which of the following statements should be made directly after the first statement "Suppose by way of contradiction that ..." ?

a) u b) p c) q d) $\neg q$ e) r

Correct Answer: q

8. Same statements as last question. In the proof that there are infinitely many prime numbers what is the final contradiction?

a) $p \wedge \neg p$ b) $q \wedge \neg q$ c) $u \wedge \neg u$ d) $p \wedge \neg q$ e) $q \wedge \neg r$

Correct Answer: $u \wedge \neg u$

9. Suppose G is the domineering game below, where X indicates that a square cannot be used.

	X	X
	X	X
	X	X

Assuming both players play randomly, so that any move is equally likely, find the probabilities: $P(Lwpf)$ and $P(Lwps)$. What is the sum $P(Lwpf) + P(Lwps)$?

a) $\frac{3}{2}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) $\frac{5}{4}$ e) 1

Correct Answer: $\frac{3}{2}$

10. Same Domineering game G as in the previous question. If we flip a fair coin to see who plays first, and then players play randomly, what is the probability that Left will win?

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{2}{3}$ d) $\frac{1}{4}$ e) 1

Correct Answer: $\frac{3}{4}$