MAT 258 Final Exam Answer Sheet

Summer 2023

Quiz ID: ZFM Name: _____

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Do your own work, and submit electronic answers at:

http://azrael.digipen.edu/cgi-bin/MAT258quiz.pl

Also submit pdf of your scratch work, your answer sheet, and your work on problems 21 and 22, on Moodle.

All submissions are due by midnight on Wednesday, July 19.

No late work or answers will be accepted.

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MAT 258 **Final Exam** Summer 2023

Multiple choice questions are worth four points each.

1. How many nonisomorphic graphs are there with four vertices and three edges?
   a) 4 b) 5 c) 2 d) 1 e) 3

2. How many injective (1-1) functions are there from the set $\{1, 2, 3\}$ to the set $\{a, b, c, d, e\}$?
   a) 125 b) 27 c) 60 d) 120 e) 32

3. Let G be the graph with vertex set $V = \{v_1, v_2, v_3, v_4, v_5\}$ and edge set

   $$E = \{\{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_3, v_5\}, \{v_4, v_5\}\}.$$

   How many cycles of length three does this graph G contain as subgraphs?
   a) 3 b) 1 c) 5 d) 2 e) 4

4. Same graph G as in the previous problem. If the edge $\{v_4, v_5\}$ is removed, what is the degree seqence of the resulting graph, given in nondecreasing order?
   a) $1, 1, 2, 3, 4$ b) $2, 2, 3, 3, 4$ c) $2, 2, 2, 2, 4$ d) $1, 1, 2, 2, 4$ e) $1, 2, 3, 3, 4$

5. How many solutions does the equation $x_1 + x_2 + x_3 + x_4 = 10$ have with integers $x_i \geq 0$?
   a) 308 b) 262 c) 250 d) 286 e) 380

6. How many solutions does the equation $x_1 + x_2 + x_3 + x_4 = 10$ have with integers $x_i \geq 0$ with the additional restrictions: $x_1 \leq 3$ and $x_2 \leq 4$?
   a) 145 b) 175 c) 150 d) 140 e) 170

7. How many strings containing only digits 0,1, or 2, contain exactly two 0's, three 1's, and four 2's ?
   a) 1260 b) 1440 c) 1184 d) 1024 e) 1320

8. Suppose that the converse of the implication $P \to Q$ is True and we also know that Q is True. What can we say about P? Choose the most accurate statement:
   a) must be T b) none of these c) could be F or T d) must be F e) is undefined

9. Suppose that the contrapositive of the implication $P \to Q$ is True and we also know that Q is True. What can we say about P? Choose the most accurate statement:
   a) must be T b) none of these c) could be F or T d) must be F e) is undefined

10. Which of the following is equivalent to the proposition $\neg P \vee Q$?
    a) $P \vee Q$ b) $P \wedge Q$ c) $Q \to P$ d) $P \to Q$ e) $\neg(P \wedge Q)$

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11. How many different alphabetized (unlabelled) strings using one or two blanks in place of one or two letters can be formed from the word VACCINE? (Two blanks can be used to replace two of the same letter or two different letters.)

a) 24 b) 25 c) 23 d) 22 e) 21

12. How many different draws from a scrabble bag will result in the ability to spell the word VACCINE assuming that none of the tiles is a blank? (Take into account the frequency of each letter, as in the file scrabble-bag.txt on the website.)

a) 13664 b) 15664 c) 14664 d) 12664 e) 11664

13. If two fair dice are rolled, let x be the number on die number 1 and y be the number on die number 2, and (x, y) be a point in the plane with distance $D = \sqrt{x^2 + y^2}$ from the origin. What is the probability that $D < 5$?

a) $\frac{13}{36}$ b) $\frac{7}{12}$ c) $\frac{1}{3}$ d) $\frac{7}{18}$ e) $\frac{5}{12}$

14. Same x, y, and D as in the previous problem. What is the probability that the distance D will be an integer?

a) $\frac{1}{36}$ b) $\frac{1}{9}$ c) $\frac{1}{6}$ d) $\frac{1}{18}$ e) $\frac{1}{12}$

15. Consider the proposition X given by $(p \to q) \to (q \to r)$. Which of the following are necessary for X to be False?

i) p is False ii) q is True iii) r is False

a) ii) and iii) only b) i) only c) i) and iii) only d) i) and ii) only e) ii) only

16. Same proposition X as in the previous problem. Suppose a bit string of length three is generated at random, and the values of the bit string are interpreted as T/F values for three propositions p, q and r, in that order. What is the probability that the proposition X is True?

a) $\frac{1}{8}$ b) $\frac{3}{8}$ c) $\frac{7}{8}$ d) $\frac{3}{4}$ e) $\frac{1}{4}$

17. Let S be the set of primes. A squarefree positive integer n is one which is not divisible by p^2 for any prime number $p \in S$. Equivalently, the prime factorization of n is a product of prime powers $p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ with all exponents $e_i = 1$ for $i = 1, \ldots, k$. Choose a logical statement that is equivalent to the *negation* of the claim: "All positive integers are squarefree". (Unless otherwise specified, assume that quantifiers range over the positive integers.)

a) $\forall p \in S : \exists n : p|n$ b) $\exists n : \forall p \in S : p|n$ c) $\exists p \in S : \forall n : p^2|n$ d) $\exists n : \exists p \in S : p^2|n$ e) $\forall p \in S : \forall n : p^2|n$

18. Suppose G is the domineering game below, where X indicates that a square cannot be used.

		X
X		
X		X

Assuming both players play randomly, so that any move is equally likely, what is the probability that Right wins playing first $P(\text{Rwpf})$?

a) $\frac{1}{4}$ b) $\frac{3}{5}$ c) $\frac{3}{4}$ d) $\frac{1}{2}$ e) $\frac{2}{3}$

19. Same Domineering game G as in the previous question, but now assume deterministic play, so that each player can choose the best move possible at any point. A winning move by player X is defined as a move which can be followed by a forced win by the player X. How many winning moves does Left have at the start of the game, assuming Left will play first?

a) 2 b) 5 c) 1 d) 3 e) 4

20. Same Domineering game G as in the previous question. How many winning moves does Right have at the start of the game, assuming Right will play first?

a) 2 b) 5 c) 1 d) 3 e) 4

The next two questions are worth 10 points each. Your work on these problems should be turned in on Moodle and should include any code used and a pdf showing your work.

21. Suppose you draw seven tiles from the scrabble bag and you notice that one of them is a blank, and the other six are non-blank. What is the probability that you can make a seven letter word, with only this additional information that exactly one of the tiles is blank? Hints: First state this as a conditional probability, so that you know what you need to compute as a fraction with numerator and denominator. Then compute the appropriate numerator using any of the code or text files that you used in the project, or any additional calculations.

22. Repeat the previous question for the case that you have exactly two blanks and five other non-blank tiles.