Homework 1 Solutions

MAT 258 — Spring 2021

Section 6.1:

1. a) 5850, b) 343

3. a) $4^{10} = 1,048,576$, b) $5^{10} = 9,765,625$.

7. $26^3 = 17,576$.

11. $2^8 = 256$.

15. $26 + 26^2 + 26^3 + 26^4 = 475,254$.

17. $128^5 - 127^5 = 1,321,368,961$.

21. a) 7, b) 5, c) 1.

35. a) 0, b) $5! = 120$, c) $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot = 720$, d) $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot = 2520$

41. n even: $2^{n/2}$, n odd: $2^{\frac{n+1}{2}}$.

51. $2^7 + 2^8 - 2^5 = 352$.

53. 107 (3 consecutive zeros) plus 48 (4 consecutive ones) minus 8 (both) equals 147.

Section 6.2:

3. a) 3, b) 14

5. 85

7. use remainders mod 4: $0, 1, 2, 3$ so of 5 there must be two with same remainder.

11. $50 \cdot 99 = 4950$, so need 4951.

17. 4.

25. (done in class) Use pigeons as the groups of consecutive boys (must be bb or b) and holes as the gaps between groups of consecutive girls (must be at least gg). There are at least 13 pigeons and at most 12 holes, so at least one hole must have 2 pigeons which gives consecutive bbb.

33. Count 3 initials: $26^3 = 17576$, multiply by 366 days, gives $6,432,816$ cases. Then take $39,000,000$ (number of people) and divide by cases, and take the ceiling function to get 7.

42. If a party has $n \geq 2$ people then there are n possible values $0, \ldots, n-1$ for the number of *other* people that are known by any one person (they cannot know themselves). But if each person knows a different number of others, then someone knows nobody and someone knows all the others, which is impossible. So at least two people must know the same number of others.