

MAT 258

Midterm Exam

Summer 2021

1. How many bit strings are there of length 6 with exactly 3 ones?

- a) 2^6 b) 20 c) 6^2 d) 32 e) 48

Correct Answer: 20

2. How many bit strings are there of length 6 which begin or end with a 1?

- a) 2^6 b) 20 c) 6^2 d) 32 e) 48

Correct Answer: 48

3. How many functions are there from the set $X = \{A, B, C\}$ to the set $Y = \{1, 2, 3, 4\}$?

- a) 35 b) 64 c) 54 d) 76 e) 81

Correct Answer: 64

4. How many one-to-one functions are there from the set $X = \{A, B, C\}$ to the set $Y = \{1, 2, 3, 4\}$?

- a) 35 b) 24 c) 16 d) 21 e) 34

Correct Answer: 24

5. How many bit strings of length 8 have exactly 4 zeros, and every occurrence of a zero followed by a one?

- a) 6 b) 3 c) $8!/4!$ d) 1 e) $4!$

Correct Answer: 1

6. How many bit strings of length 8 have less than 4 zeros, and every occurrence of a zero followed by a one?

- a) 33 b) 36 c) 39 d) 30 e) 42

Correct Answer: 33

7. Of eight players, five are chosen to be starters on a soccer team. In how many different ways can this assignment be made?

- a) $\frac{8!}{5!}$ b) $\frac{8 \cdot 7 \cdot 6}{3!}$ c) $\frac{8!}{4!}$ d) $8 \cdot 7 \cdot 6$ e) $8 \cdot 7 \cdot 6 \cdot 5$

Correct Answer: $\frac{8 \cdot 7 \cdot 6}{3!}$

8. Find the coefficient of a^4b^5 in the expansion $(a + b)^9$:

- a) 120 b) 126 c) 116 d) 100 e) 110

Correct Answer: 126

9. Evaluate the following sum using the binomial theorem where n is *any* positive integer:

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{n-1} + \binom{n}{n}.$$

- a) 1 b) 0 c) -1 d) 2^n e) 2^{n-1}

Correct Answer: 2^n

10. Evaluate the alternating sum (using the binomial theorem) where n is an *odd* integer:

$$\binom{n}{0} - 2\binom{n}{1} + 2^2\binom{n}{2} - 2^3\binom{n}{3} + \cdots + 2^{n-1}\binom{n}{n-1} - 2^n\binom{n}{n}.$$

- a) 1 b) 0 c) -1 d) 2^n e) 2^{n-1}

Correct Answer: -1

11. Let i and m be integers with $1 \leq i \leq m$. Let A be a set with m elements, let $a \in A$, and let $B = A \setminus \{a\}$ (ie. all elements of A except a .) Let j be the number of subsets of B of size $i-1$, and let k be number of subsets of B of size i . Which of the following is equivalent to $j+k$?

- a) 0 b) 2^m c) 2^{m-1} d) $\binom{m+1}{i+1}$ e) $\binom{m}{i}$

Correct Answer: $\binom{m}{i}$

12. How many solutions does the equation $x_1 + x_2 + x_3 + x_4 = 9$ have with integers $x_i \geq 0$?

- a) 220 b) 210 c) 200 d) 180 e) 160

Correct Answer: 220

13. How many solutions does the equation $x_1 + x_2 + x_3 + x_4 = 9$ have with integers $x_i \geq 0$ with the additional restrictions: $x_1 \geq 2$ and $x_3 \geq 3$?

- a) 40 b) 35 c) 45 d) 50 e) 75

Correct Answer: 35

14. How many solutions does the equation $x_1 + x_2 + x_3 + x_4 = 9$ have with integers $x_i \geq 0$ with the additional restrictions: $x_1 \leq 2$ and $x_3 \leq 3$?

- a) 100 b) 70 c) 80 d) 90 e) 60

Correct Answer: 90

15. How many strings containing only digits 0,1, or 2, contain exactly two 0's, two 1's, and two 2's ?

- a) 82 b) 84 c) 90 d) 86 e) 88

Correct Answer: 90

16. In lexicographic order, which permutation comes directly after 54231 in the list of all permutations of the numbers 1,2,3,4,5 ?

- a) 54213 b) 54123 c) 53421 d) 54312 e) 53214

Correct Answer: 54312

17. How many subsets of the set $\{1, 2, 3, 4, 5, 6, 7\}$ contain at least two elements?

- a) 72 b) 327 c) 234 d) 172 e) 120

Correct Answer: 120

18. How many bit strings of length 7 contain the substring 1011 ?
a) 30 b) 34 c) 31 d) 32 e) 33
Correct Answer: 31
19. How many bit strings of length 7 contain the substring 1011 *or* the substring 0111? (Hint: add to those in the previous question the strings which contain 0111 but not 1011.)
a) 51 b) 54 c) 52 d) 50 e) 53
Correct Answer: 50
20. A task consists of lining up 5 people from left to right and then placing a penny, a nickel, a dime, and a quarter into the left or right pocket of 4 of these 5 people. In the end there will be exactly one coin in exactly one pocket of exactly 4 of the 5 people, who are standing in order from left to right. In how many different ways can this task be performed?
a) $(6!)^2 4!$ b) $(5!)^2 2^4$ c) $(5!4!)^2$ d) $(5!2^4)^2$ e) $(5!)^2 2^3$
Correct Answer: $(5!)^2 2^4$
21. How many different alphabetized (unlabelled) strings using one or two blanks in place of one or two letters can be formed from the word BENZENE? (Two blanks can be used to replace two of the same letter or two different letters.)
a) 12 b) 13 c) 14 d) 16 e) 15
Correct Answer: 12
22. How many different draws from a scrabble bag will result in the ability to spell the word BENZENE assuming that none of the tiles is a blank? (Take into account the frequency of each letter, as in the file scrabble-bag.txt on the website.)
a) 5600 b) 4600 c) 3600 d) 6600 e) 7600
Correct Answer: 6600
23. If two fair dice are rolled, what is the probability that the product of the two numbers is a multiple of 6?
a) $\frac{3}{4}$ b) $\frac{1}{3}$ c) $\frac{5}{12}$ d) $\frac{5}{9}$ e) $\frac{7}{12}$
Correct Answer: $\frac{5}{12}$
24. If two fair dice are rolled, what is the probability that the product of the two numbers is a multiple of 6, given that at least one of the numbers rolled is even.
a) $\frac{3}{4}$ b) $\frac{1}{3}$ c) $\frac{5}{12}$ d) $\frac{5}{9}$ e) $\frac{7}{12}$
Correct Answer: $\frac{5}{9}$
25. When two dice are rolled, assume one of them is biased, so that a one showing up on that die has a probability of $\frac{1}{2}$, and all other numbers on that die are equally likely. Assume the other die is fair and that the roll of each die is independent of the other one. Find the probability that the sum on the two dice equals seven.
a) $\frac{7}{36}$ b) $\frac{5}{21}$ c) $\frac{1}{6}$ d) $\frac{1}{12}$ e) $\frac{5}{33}$
Correct Answer: $\frac{1}{6}$