

## MAT 300/500

## Quiz 2

## Spring 2021

1. Find the derivative:  $\frac{d}{dt}B_3^4(t)$ .  
 a)  $4t^2(3-4t)$       b)  $4t(3-4t^2)$       c)  $4t-3t^2$       d)  $3t^3-4t^2$       e)  $3t^2-4t^3$   
 Correct Answer:  $4t^2(3-4t)$
2. Choose a correct equivalent expression for  $B_3^5(t)$ :  
 a)  $tB_4^4(t) + (1-t)B_3^3(t)$       b)  $tB_1^4(t) + (1-t)B_2^4(t)$       c)  $tB_3^5(t) + (1-t)B_4^5(t)$       d)  $tB_2^4(t) + (1-t)B_3^4(t)$   
 e)  $tB_3^4(t) + (1-t)B_4^4(t)$   
 Correct Answer:  $tB_2^4(t) + (1-t)B_3^4(t)$
3. By the linearity property of determinants with respect to one row or column,  $\begin{vmatrix} 1 & 2 & 4 \\ -2 & -4 & -6 \\ 3 & 6 & 9 \end{vmatrix}$  is equivalent to:  
 a)  $-2 \begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 3 & 6 & 9 \end{vmatrix}$       b)  $-2 \begin{vmatrix} 1 & -1 & 4 \\ 1 & 2 & 3 \\ 3 & -3 & 9 \end{vmatrix}$       c)  $3 \begin{vmatrix} 1 & 2 & 4 \\ -2 & -4 & -6 \\ 1 & 2 & 3 \end{vmatrix}$       d)  $2 \begin{vmatrix} 1 & 2 & 4 \\ -2 & -4 & -6 \\ 3 & 3 & 3 \end{vmatrix}$       e)  $2 \begin{vmatrix} 1 & 1 & 4 \\ -2 & 1 & -6 \\ 3 & 1 & 9 \end{vmatrix}$   
 Correct Answer:  $3 \begin{vmatrix} 1 & 2 & 4 \\ -2 & -4 & -6 \\ 1 & 2 & 3 \end{vmatrix}$
4. The polynomial  $p(x) = \begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 1 & 5 & 25 \end{vmatrix}$  has zeros at  $x$  equal to:  
 a) 2 and 4      b) 2 and 3      c) 1 and 2      d) 3 and 5      e) 2 and 5  
 Correct Answer: 2 and 5
5. Same  $p(x)$  as in the previous question. The leading coefficient of  $p(x)$  is:  
 a) 4      b) 5      c) 1      d) 2      e) 3  
 Correct Answer: 3
6. If an interpolating polynomial  $p(t) \in P_2$  has values that match  $g(2) = 0$  and  $g(5) = 0$  and  $g(0) = 30$  (which is true for the polynomial in the previous problem), then what is the bracket operator  $[0, 2, 5]g$ ? (Hint: Use the definition of the bracket operator.)  
 a) 4      b) 5      c) 1      d) 2      e) 3  
 Correct Answer: 3
7. Use a simple observation (about the graph) to find the interpolating polynomial in  $P_2$  that passes through the points  $(1, 2)$ ,  $(2, 1)$ , and  $(4, -1)$ . In the standard basis, with  $p(t) = a_0 + a_1t + a_2t^2$ , what is the coefficient  $a_2$ ?  
 a)  $-1$       b)  $-\frac{1}{2}$       c)  $\frac{1}{2}$       d) 1      e) 0  
 Correct Answer: 0
8. Same  $p(t)$  as in the previous question. What is the coefficient  $a_1$ ?  
 a)  $-1$       b)  $-\frac{1}{2}$       c)  $\frac{1}{2}$       d) 1      e) 0  
 Correct Answer:  $-1$
9. Compute the divided difference  $[0, 1, 2]g$  if  $g(0) = 2$ ,  $g(1) = -1$  and  $g(2) = 0$ .  
 a)  $-1$       b)  $-\frac{1}{2}$       c)  $\frac{1}{2}$       d) 1      e) 0  
 Correct Answer: 2

10. Below is an interpolating polynomial written with Lagrange polynomials, that passes through the points:  $(0, 2)$ ,  $(1, 4)$ ,  $(3, -3)$ . Find the correct value of the missing constant  $C$ :

$$(2) \frac{(t-1)(t-3)}{(0-1)(0-3)} + (4) \frac{(t-0)(t-3)}{(1-0)(1-3)} + (-3) \frac{(t-0)(t-1)}{C}$$

- a)  $-12$                       b)  $-6$                       c)  $12$                       d)  $6$                       e)  $3$

Correct Answer:  $6$