Test ID: TESTID Name: ____________________

MAT 300 **Quiz 3** Fall 2019

1. A nonzero polynomial $p(t)$ of degree d has the properties: $p(1) = p'(1) = 0$, $p(2) = p'(2) = p''(2) = 0$. Which of the following must be true?

   i) $(t-1)^2$ is factor of $p(t)$ ii) $(t-2)^2$ is factor of $p(t)$ iii) $d \geq 5$

   a) i) only b) ii) only c) all of them d) i) and ii) only e) i) and iii) only

   Correct Answer: all of them

2. If $p(t) = 2 + 3(t-1) + 4(t-1)^2$ is the Newton form of the osculating polynomial that matches a data function $g(t)$ for data values $t_0 = 1$, $t_1 = 1$, and $t_2 = 2$, what is $[1, 1, 2]g$?

   a) 2 b) 3 c) 4 d) 5 e) 6

   Correct Answer: 4

3. Same $p(t)$ and $g(t)$ as in the previous question. What is $[1, 1]g$?

   a) 2 b) 3 c) 4 d) 5 e) 6

   Correct Answer: 3

4. Let $F_5[u_1, u_2, u_3, u_4, u_5]$ be the polar form of the polynomial t^5 in P_5. Find $F_5[0, 1, 1, 1, 1]$.

   a) 0 b) 1 c) $\frac{3}{5}$ d) $\frac{1}{2}$ e) $\frac{1}{5}$

   Correct Answer: 0

5. Let $F_2[u_1, u_2, u_3, u_4, u_5]$ be the polar form of the polynomial t^2 in P_5. Find $F_2[0, 1, 1, 1, 1]$.

   a) 0 b) 1 c) $\frac{3}{5}$ d) $\frac{1}{2}$ e) $\frac{1}{5}$

   Correct Answer: $\frac{3}{5}$

6. Let $\gamma(t) = (t^2 - t^5, t^2 + t^5)$. Use the polar form of $\gamma(t)$ with respect to the standard basis to find the control point P_4.

   a) $(\frac{3}{5}, \frac{3}{5})$ b) $(\frac{1}{2}, \frac{1}{2})$ c) $(\frac{1}{5}, \frac{1}{5})$ d) $(1, 1)$ e) $(2, 2)$

   Correct Answer: $(\frac{3}{5}, \frac{3}{5})$

7. Let $g(t) = (1-t)^2 t^2$, and let $G[u_1, u_2, u_3, u_4] = \frac{1}{2}[(1-u_1)(1-u_2)u_3u_4 + (1-u_3)(1-u_4)u_1u_2]$. For G to be the polar form of g it would need to satisfy the three defining properties of polar forms: i) symmetry, ii) substitution, and iii) affine. Determine if it is True or False that G satisfies these properties. Answers are in order i),ii),iii).

   a) TFT b) TTT c) FTT d) FFF e) TTF

   Correct Answer: FTT

8. Let f be the function whose graph is the piecewise linear path connecting the points: $(0, 1)$, $(1, 2)$, $(2, 0)$, and $(3, 0)$. Then f is a member of the vector space $V = P^3_{1,0}[0, 1, 2, 3]$, with basis $\{1, t, (t-1)^1_+, (t-2)^1_+\}$. Write $f(t) = a_0 + a_1 t + a_2(t-1)^1_+ + a_3(t-2)^1_+$. What is a_2?

   a) 2 b) -3 c) 4 d) 3 e) -2

   Correct Answer: -3

9. Same function f and vector space V as in the previous question. What is a_3?

   a) 2 b) -3 c) 4 d) 3 e) -2

   Correct Answer: 2

10. Same function f and vector space V as in the previous question. Let $h_0(t)$ be $1-t$ for $0 \leq t < 1$ and zero elsewhere. Let $h_1(t)$ be t for $0 \leq t < 1$, $2-t$ for $1 \leq t < 2$, and zero elsewhere. Let $h_2(t)$ be $t-1$ for $1 \leq t < 2$, $3-t$ for $2 \leq t \leq 3$, and zero elsewhere. Let $h_3(t)$ be $t-2$ for $2 \leq t \leq 3$ and zero elsewhere. Then $\{h_0(t), h_1(t), h_2(t), h_3(t)\}$ restricted to $[0,3]$ is also a basis of V. If we write $f(t) = c_0h_0(t) + c_1h_1(t) + c_2h_2(t) + c_3h_3(t)$, then what is c_1?

a) 2 b) -3 c) 4 d) 3 e) -2

Correct Answer: 2