Test ID: TESTID Name: ____________________

MAT 300/500 **Quiz 3** Fall 2021

1. A nonzero polynomial $p(t)$ of degree d has the properties: $p(1) = p'(1) = 0$, $p(2) = p'(2) = p''(2) = 0$. Which of the following must be true?

   i) $(t-1)^2$ is factor of $p(t)$ ii) $(t-2)^2$ is factor of $p(t)$ iii) $d \geq 5$

   a) i) only b) ii) only c) all of them d) i) and ii) only e) i) and iii) only

   Correct Answer: all of them

2. Let $F_5[u_1, u_2, u_3, u_4, u_5]$ be the polar form of the polynomial t^5 in P_5. Find $F_5[0, 1, 1, 1, 1]$.

   a) 0 b) 1 c) $\frac{3}{5}$ d) $\frac{1}{2}$ e) $\frac{1}{5}$

   Correct Answer: 0

3. Let $F_2[u_1, u_2, u_3, u_4, u_5]$ be the polar form of the polynomial t^2 in P_5. Find $F_2[0, 1, 1, 1, 1]$.

   a) 0 b) 1 c) $\frac{3}{5}$ d) $\frac{1}{2}$ e) $\frac{1}{5}$

   Correct Answer: $\frac{3}{5}$

4. Let $\gamma(t) = (t^2 - t^5, t^2 + t^5)$. Use the polar form of $\gamma(t)$ with respect to the standard basis to find the control point P_4.

   a) $(\frac{3}{5}, \frac{3}{5})$ b) $(\frac{1}{2}, \frac{1}{2})$ c) $(\frac{1}{5}, \frac{1}{5})$ d) $(1, 1)$ e) $(2, 2)$

   Correct Answer: $(\frac{3}{5}, \frac{3}{5})$

5. Let $g(t) = (1-t)^2 t^2$, and let $G[u_1, u_2, u_3, u_4] = \frac{1}{2}[(1-u_1)(1-u_2)u_3u_4 + (1-u_3)(1-u_4)u_1u_2]$. For G to be the polar form of g it would need to satisfy the three defining properties of polar forms: i) symmetry, ii) substitution, and iii) affine. Determine if it is True or False that G satisfies each of these properties. Answers are in order i),ii),iii).

   a) TFT b) FTT c) FTF d) FFF e) TTF

   Correct Answer: FTT

6. Let $F[u_1, u_2]$ be the polar form of a quadratic Bezier curve $\gamma(t)$ with control points $P_0 = (0, 1)$, $P_1 = (1, 2)$, and $P_2 = (-1, 5)$. Use Nested Linear Interpolation to find $F[1, 2]$:

   a) $(-3, 8)$ b) $(3, 4)$ c) $(8, 3)$ d) $(4, -3)$ e) $(5, -4)$

   Correct Answer: $(-3, 8)$

7. Suppose f, g, and h are functions with $f = gh$. Suppose also that p_f, p_g, and p_h are interpolating polynomials in P_2 for the data $t_0 = -1$, $t_1 = 2$, $t_2 = 3$ and for each data function, so that $p_f(t_i) = f(t_i)$, $p_g(t_i) = g(t_i)$, and $p_h(t_i) = h(t_i)$, for $i = 0, 1, 2$. Let $q(t) = p_g(t)p_h(t)$. True or False:

   i) $q(t_i) = f(t_i)$, $i = 0, 1, 2$ ii) $q(t) \in P_2$ iii) $q(t) = p_f(t)$

   a) TFT b) FFT c) FTT d) TFF e) TTT

   Correct Answer: TFF

8. Same functions as in previous question. Now also suppose that $q(t) = F(t) + G(t)$ where $G(t)$ consists of terms from the product $p_g(t)p_h(t)$ which contain each of the factors $t - t_i$, for $i = 0, 1, 2$. With this assumption, determine if the statements are True or False:

   i) $G(t_i) = 0$, $i = 0, 1, 2$ ii) $F(t) \in P_2$ iii) $F(t) = p_f(t)$

   a) TFT b) FFT c) FTT d) TFF e) TTT

   Correct Answer: TTT