

MAT 300/500

Quiz 4

Spring 2021

1. Let f be the piecewise polynomial function defined by:

$$f(t) = \begin{cases} t^2 - 1, & 0 \leq t < 1 \\ (t - 2)^2 - 1, & 1 \leq t < 3 \\ 1 - (t - 4)^2, & 3 \leq t \leq 4 \end{cases}$$

True or False: answers are in the order i,ii,iii

- i) f is differentiable at $t = 1$ ii) f is differentiable at $t = 2$ iii) f is differentiable at $t = 3$
 a) FFF b) FTT c) TTF d) FFT e) TTT

Correct Answer: FTT

2. Same f as in the previous question. True or False: answers are in the order i,ii,iii

- i) f is in $P_{2,0}^3[0, 1, 3, 4]$ ii) f is in $P_{2,\mathbf{r}}^3[0, 1, 3, 4]$, $\mathbf{r} = (0, 1)$ iii) f is in $P_{2,\mathbf{r}}^3[0, 1, 3, 4]$, $\mathbf{r} = (1, 0)$
 a) TTF b) FTF c) TTT d) TTF e) FFT

Correct Answer: TTF

3. Same f as in the previous question. If f is written as:

$$f(t) = (t - 0)_+^2 - (t - 0)_+^0 + a(t - 1)_+^2 + b(t - 3)_+^1 + c(t - 3)_+^2$$

find the correct coefficient a : (Hint: on the appropriate subinterval, first equate the coefficients of t^2 .)

- a) -4 b) -2 c) 4 d) 2 e) 0

Correct Answer: 0

4. Same form for f as in the previous question. Find the correct coefficient b :

- a) -4 b) -2 c) 2 d) 4 e) 0

Correct Answer: -4

5. Same form for f as in the previous question. Find the correct coefficient c : (Hint: on the appropriate subinterval, first equate the coefficients of t^2 .)

- a) -4 b) -2 c) 2 d) 4 e) 0

Correct Answer: -2

6. What is the dimension of $P_{4,r}^5[-3, -2, -1, 0, 1, 2]$, if $r = 1$?

- a) 17 b) 15 c) 14 d) 16 e) 13

Correct Answer: 17

7. What is the dimension of $P_{4,\mathbf{r}}^5[-3, -2, -1, 0, 1, 2]$, if $\mathbf{r} = (1, 2, 3, 0)$?

- a) 18 b) 15 c) 14 d) 16 e) 13

Correct Answer: 15

8. Find the intersection $V \cap W$ of the vector spaces $V = P_{4,\mathbf{r}}^5[-3, -2, -1, 0, 1, 2]$ with $\mathbf{r} = (3, 2, 1, 0)$, and $W = P_{4,\mathbf{r}}^5[-3, -2, -1, 0, 1, 2]$ with $\mathbf{r} = (0, 1, 2, 3)$. What is the dimension of this intersection?

- a) 12 b) 14 c) 13 d) 10 e) 11

Correct Answer: 11

9. Assume $a < b$. Let $f(t) = [a, b](t - x)_+^3$, where the divided difference is computed with x as the variable, and t as constant. Find $f(\frac{a+b}{2})$:

- a) $b - a$ b) 0 c) $-\frac{(b-a)^2}{8}$ d) $\frac{b-a}{8}$ e) $\frac{(b-a)^3}{2}$

Correct Answer: $-\frac{(b-a)^2}{8}$

10. Same $f(t)$ as in the previous question. Find $f(a - 1)$:

- a) $b - a$ b) 0 c) $-\frac{(b-a)^2}{8}$ d) $\frac{b-a}{8}$ e) $\frac{(b-a)^3}{2}$

Correct Answer: 0