

MAT 300/500

Quiz 4

Spring 2024

1. Let f be the piecewise polynomial function defined by:

$$f(t) = \begin{cases} t^2 - 1, & 0 \leq t < 1 \\ (t-2)^2 - 1, & 1 \leq t < 3 \\ 1 - (t-4)^2, & 3 \leq t \leq 4 \end{cases}$$

True or False: answers are in the order i,ii,iii

- i) f is differentiable at $t = 1$ ii) f is differentiable at $t = 2$ iii) f is differentiable at $t = 3$
 a) FFF b) FTT c) TTF d) FFT e) TTT

Correct Answer: FTT

2. Same f as in the previous question. True or False: answers are in the order i,ii,iii

- i) f is in $P_{2,0}^3[0, 1, 3, 4]$ ii) f is in $P_{2,\mathbf{r}}^3[0, 1, 3, 4]$, $\mathbf{r} = (0, 1)$ iii) f is in $P_{2,\mathbf{r}}^3[0, 1, 3, 4]$, $\mathbf{r} = (1, 0)$
 a) TFT b) FTF c) TTT d) TTF e) FFT

Correct Answer: TTF

3. Same f as in the previous question. If f is written as:

$$f(t) = t^2 - 1 + a(t-1)_+^2 + b(t-1)_+^1 + c(t-3)_+^2$$

find the correct coefficient a : (Hint: restricted to the subinterval $[1, 3)$, first equate the coefficients of t^2 on both sides of the above equation.)

- a) -4 b) -2 c) 4 d) 2 e) 0

Correct Answer: 0

4. Same form for f as in the previous question. Find the correct coefficient b :

- a) -4 b) -2 c) 2 d) 4 e) 0

Correct Answer: -4

5. Same form for f as in the previous question. Find the correct coefficient c :

- a) -4 b) -2 c) 2 d) 4 e) 0

Correct Answer: -2

6. What is the dimension of $P_{4,\mathbf{r}}^5[-3, -2, -1, 0, 1, 2]$, if $\mathbf{r} = 1$?

- a) 17 b) 15 c) 14 d) 16 e) 13

Correct Answer: 17

7. What is the dimension of $P_{4,\mathbf{r}}^5[-3, -2, -1, 0, 1, 2]$, if $\mathbf{r} = (1, 2, 3, 0)$?

- a) 17 b) 15 c) 14 d) 16 e) 13

Correct Answer: 15

8. Find the intersection $V \cap W$ of the vector spaces $V = P_{4,\mathbf{r}}^5[-3, -2, -1, 0, 1, 2]$ with $\mathbf{r} = (3, 2, 1, 0)$, and $W = P_{4,\mathbf{r}}^5[-3, -2, -1, 0, 1, 2]$ with $\mathbf{r} = (0, 1, 2, 3)$. What is the dimension of this intersection?

- a) 12 b) 14 c) 13 d) 10 e) 11

Correct Answer: 11

9. Suppose the knot sequence $\{-1, 0, 1, 2, 2, 3, 4, 4, 4, 6, 6, 6\}$ is used to define 10 B -splines of degree 1 by using the appropriate subsequences of consecutive knots. How many of the those B -splines will be exactly zero? (Use the B -spline definition.)

- a) 3 b) 2 c) 1 d) 0 e) 4

Correct Answer: 2

10. Suppose the knot sequence $\{-2, -1, 0, 1, 2, 3, 3, 3, 3, 4, 4, 5, 5, 6, 6, 6\}$ is used to define 13 B -splines of degree 2 by using the appropriate subsequences of consecutive knots. How many of the those B -splines will be exactly zero? (Use the B -spline definition.)

- a) 3 b) 2 c) 1 d) 0 e) 4

Correct Answer: 1