

MAT 300/500 Quiz 5 Answer Sheet

Spring 2024

Quiz ID: WHK

Name: \_\_\_\_\_

1.

2.

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Submit electronic answers at

<http://azrael1.digipen.edu/cgi-bin/MAT300quiz.pl>

## MAT 300/500

## Quiz 5

## Spring 2024

- Let  $\mathbf{t}$  be the knot sequence  $\{-2, -1, 0, 1, 1, 2, 2, 2, 3, 4, 4, 4, 6, 6, 6\}$ , with associated quadratic  $B$ -splines which are a basis of a vector space  $V$  of the form  $P_{2,\mathbf{r}}^k[0, 1, 2, 3, 4, 6]$ . Suppose that  $\gamma(t) = \sum_{i=0}^{11} P_i \mathcal{B}_i^d(t)$  is a  $B$ -spline curve with control points  $P_0, \dots, P_{11}$ . To compute  $\gamma(3.5)$ , what is the index  $J$  in the DeBoor algorithm (defined by  $t \in [t_J, t_{J+1})$ )?
  - 7
  - 10
  - 8
  - 9
  - 6
- Same knot sequence  $\mathbf{t}$  and vector space  $V$  as in the previous question. Find the continuity vector  $\mathbf{r}$ .
  - $(1, 0, 0, -1)$
  - $(0, -1, 1, -1)$
  - $(2, -1, 1, 0)$
  - $(2, 3, 1, 3)$
  - $(0, 1, 0, -1)$
- Same knot sequence  $\mathbf{t}$  and vector space  $V$  as in the previous question. Find the dimension of  $V$ .
  - 13
  - 14
  - 12
  - 10
  - 11
- Same knot sequence  $\mathbf{t}$  and vector space  $V$  as in the previous question. Let  $S$  be the  $B$ -spline basis of  $V$  with knot sequence  $\mathbf{t}$ . How many  $B$ -splines in  $S$  are zero for all  $t$  in the interval  $[2, 3]$ ?
  - 8
  - 5
  - 9
  - 7
  - 6
- Same knot sequence  $\mathbf{t}$  and vector space  $V$  as in the previous question. Let  $S$  be the  $B$ -spline basis of  $V$  with knot sequence  $\mathbf{t}$ . How many  $B$ -splines in  $S$  have exact order of continuity  $r = 1$  at  $t = 2$ ? (Recall: exact order of continuity of a  $B$ -spline  $\mathcal{B}_i^d(t)$  at a knot value  $t_j$  is predicted by the multiplicity of  $t_j$  inside the sequence of knots  $t_i, \dots, t_{i+d+1}$  used to define the  $B$ -spline  $\mathcal{B}_i^d(t)$ .)
  - 3
  - 4
  - 5
  - 2
  - 1
- Same knot sequence  $\mathbf{t}$  and vector space  $V$  as in the previous question. Let  $S$  be the  $B$ -spline basis of  $V$  with knot sequence  $\mathbf{t}$ . How many  $B$ -splines in  $S$  have exact order of continuity  $r = 1$  at  $t = 3$ ?
  - 3
  - 4
  - 5
  - 2
  - 1
- Which sequence of four knots will result in a quadratic  $B$ -spline which is continuous and consists of two parabolas which meet at a point which is the only place where the function is not differentiable?
  - $0, 1, 2, 2$
  - $0, 0, 1, 1$
  - $0, 1, 1, 2$
  - $0, 0, 0, 1$
  - $0, 0, 1, 2$
- Which sequence of four knots will result in a quadratic  $B$ -spline which is continuous and differentiable except at one point, where it is continuous but not differentiable, and consists of two parabolas one of which is concave up and the other concave down?
  - $0, 1, 2, 2$
  - $0, 0, 1, 1$
  - $0, 1, 1, 2$
  - $0, 0, 0, 1$
  - $0, 0, 1, 2$
- Which sequence of four knots will result in a quadratic  $B$ -spline which is continuous and which consists of only one parabola?
  - $0, 1, 2, 2$
  - $0, 0, 1, 1$
  - $0, 1, 1, 2$
  - $0, 0, 0, 1$
  - $0, 0, 1, 2$
- Which sequence of five knots will result in a cubic  $B$ -spline which is continuous, but fails to have a continuous first derivative?
  - $0, 1, 1, 2, 2$
  - $0, 0, 0, 1, 2$
  - $0, 1, 1, 2, 3$
  - $0, 1, 1, 1, 1$
  - $0, 0, 1, 1, 2$