

MAT 300/500

Quiz 5

Spring 2021

1. Suppose the knot sequence $\{-2, -1, 0, 1, 2, 2, 3, 4, 4, 4, 6, 6, 6\}$ is used to define 11 B -splines of degree 1 by using the appropriate subsequences of consecutive knots. How many of the those B -splines will be exactly zero? (Use the B -spline definition.)

a) 3 b) 2 c) 1 d) 5 e) 4

Correct Answer: 2

2. Let \mathbf{t} be the knot sequence $\{-2, -1, 0, 1, 2, 2, 3, 4, 4, 4, 6, 6, 6\}$, with associated quadratic B -splines which are a basis of a vector space V of the form $P_{2,\mathbf{r}}^k[0, 1, 2, 3, 4, 6]$. Suppose that $\gamma(t) = \sum_{i=0}^{11} P_i \mathcal{B}_i^d(t)$ is a B -spline curve with control points P_0, \dots, P_{11} . To compute $\gamma(3.5)$, what is the index J in the DeBoor algorithm (defined by $t \in [t_J, t_{J+1})$)?

a) 9 b) 6 c) 7 d) 8 e) 10

Correct Answer: 6

3. Same knot sequence \mathbf{t} and vector space V as in the previous question. Find the continuity vector \mathbf{r} .

a) (1, 3, 3, 2) b) (-1, 1, 0, -1) c) (1, -1, 0, -1) d) (2, 1, 1, 0) e) (1, 0, 1, -1)

Correct Answer: (1, 0, 1, -1)

4. Same knot sequence \mathbf{t} and vector space V as in the previous question. Find the dimension of V .

a) 10 b) 11 c) 13 d) 12 e) 14

Correct Answer: 10

5. Same knot sequence \mathbf{t} and vector space V as in the previous question. Let S be the B -spline basis of V . How many B -splines in S are zero for all t outside of an interval of the form $[i, i + 1]$ for some integer i ?

a) 2 b) 1 c) 3 d) 5 e) 4

Correct Answer: 4

6. Same knot sequence \mathbf{t} and vector space V as in the previous question. Let S be the B -spline basis of V . How many B -splines in S have exact order of continuity $r = 0$ at $t = 2$?

a) 2 b) 1 c) 3 d) 5 e) 4

Correct Answer: 3

7. Same knot sequence \mathbf{t} and vector space V as in the previous question. Let S be the B -spline basis of V . Suppose a sequence of inputs is chosen $0 < v_0 < v_1 < \dots < v_{n-1} < 6$, with n equal to the dimension of V . Call this sequence *good* if it satisfies the criterion of the Schoenberg-Whitney Theorem, guaranteeing that there will be a unique f in V which interpolates the data inputs with any chosen outputs. Which values for v_4, v_5, v_6 will guarantee that the sequence fails to be good?

a) 2.2, 2.8, 2.9 b) 2.1, 2.3, 3.4 c) 2.5, 3.6, 3.7 d) 2.1, 2.9, 3.1 e) 2.8, 2.9, 3.1

Correct Answer: 2.2, 2.8, 2.9

8. Which sequence of five knots will result in a cubic B -spline which is differentiable, but fails to have a continuous second derivative?

a) 0, 1, 1, 1, 1 b) 0, 1, 1, 1, 2 c) 0, 1, 2, 2, 2 d) 0, 1, 1, 2, 3 e) 0, 0, 0, 1, 2

Correct Answer: 0, 1, 1, 2, 3