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MAT 300/500 **Quiz 6** Spring 2026

Recall some important facts:

- knot sequences are non-decreasing: $t_i \leq t_{i+1}$
- $\mathcal{B}_i^d(t)$ uses only the consecutive knot values $t_i, \ldots, t_{i+d+1}$.
- $\mathcal{B}_i^d(t) > 0$ for $t_i < t < t_{i+d+1}$, and zero for $t < t_i$ and $t > t_{i+d+1}$.
- $\mathcal{B}_i^d(t) > 0$ has order of continuity $r = d - m$ at $t = t_j$ $(i \leq j \leq i + d + 1)$ where m is the multiplicity of t_j in the sequence $t_i, \ldots, t_{i+d+1}$.
- A knot sequence $\mathbf{t}$ for a B-spline basis of a vector space $V = P_{d,\mathbf{r}}^k[u_0, \ldots, u_k]$ consists of 3 distinct parts: $d+1$ values $t_0, \ldots, t_d \leq u_0$, then break-points with multiplicities (all break-points u_i satisfy $u_0 < u_i < u_k$), and $d+1$ values $t_{N-d}, \ldots, t_N$ which are all $\geq u_k$.

1. Let $\mathbf{t} = \{-1, -1, 0, 0, 0, 0, 0, 1, 1, 1, 2, 2, 3, 4, 4, 4, 4, 5, 5, 5, 6\}$. How many non-zero B-splines of degree $d = 2$ are attached to this knot sequence?

   a) 11 b) 12 c) 13 d) 14 e) 15

   Correct Answer: 15

2. Same $\mathbf{t}$ as in the previous question. What is the minimum number of knot values that should be deleted from $\mathbf{t}$ to form a new knot sequence $\mathbf{t}'$ so that all the B-splines of degree 2 associated to $\mathbf{t}'$ are non-zero?

   a) 1 b) 3 c) 2 d) 5 e) 4

   Correct Answer: 3

3. Same $\mathbf{t}'$ as in the previous question. What is the minimum number of knot values that should be added to the ends of $\mathbf{t}'$ to form a new knot sequence $\mathbf{t}''$ so that the B-splines of degree 2 associated to $\mathbf{t}''$ are a basis of a vector space $V = P_{2,\mathbf{r}}^k[u_0, \ldots, u_k]$? Only add the new knot values on the end, on one or both sides of $\mathbf{t}'$ and do this with the minimal number of knots possible.

   a) 1 b) 2 c) 5 d) 3 e) 4

   Correct Answer: 3

4. Assume that any degree 1 B-spline $\mathcal{B}_i^1(t)$ with knot values $t_i < t_{i+1} < t_{i+2}$ (no repeated knots) is a continuous hat function with value 1 at $t = t_{i+1}$ and value zero at t_i and t_{i+2}. Also assume the derivative formula for B-splines:

$$\frac{d}{dt}\mathcal{B}_i^d(t) = d\left(\frac{\mathcal{B}_i^{d-1}(t)}{t_{i+d} - t_i} - \frac{\mathcal{B}_{i+1}^{d-1}(t)}{t_{i+d+1} - t_{i+1}}\right)$$

   Let $f(t) = \mathcal{B}_0^2(t)$ be the quadratic $(d = 2)$ B-spline with knot sequence $0, 1, 2, 3$. Find $f'(2)$:

   a) 0 b) 1 c) $\frac{1}{2}$ d) $-\frac{1}{2}$ e) -1

   Correct Answer: -1

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5. Let $f(t) = \mathcal{B}_0^2(t)$ be the quadratic ($d = 2$) B-spline with knot sequence $0, 1, 2, 3$. Find $f'(1.5)$:

a) 0 b) 1 c) $\frac{1}{2}$ d) $-\frac{1}{2}$ e) -1

Correct Answer: 0

6. Let $\mathbf{t}$ be the knot sequence $\{-2, -1, 0, 1, 1, 2, 2, 3, 3, 3, 4, 4, 6, 6, 6\}$, with associated quadratic ($d = 2$) B-splines which are a basis of a vector space V of the form $P_{2,\mathbf{r}}^k[0, 1, 2, 3, 4, 6]$, with dimension n. Suppose that $\gamma(t) = \sum_{i=0}^{n-1} P_i \mathcal{B}_i^2(t)$ is a B-spline curve with control points $P_0, \ldots, P_{n-1}$. Suppose that the index J in the DeBoor algorithm is 6 in order to compute $\gamma(t)$ for some t value in the interval $(0, 6)$. Which t-value could correspond to this J?

a) 1.2 b) 0.3 c) 3.5 d) 2.1 e) 4.3

Correct Answer: 2.1

7. Same knot sequence $\mathbf{t}$ and vector space V as in the previous question. Find the minimum required order of continuity r at $t = 2$:

a) 1 b) -1 c) 0 d) 2 e) 3

Correct Answer: 0

8. Same knot sequence $\mathbf{t}$ and vector space V as in the previous question. Find the minimum required order of continuity r at $t = 3$:

a) 1 b) -1 c) 0 d) 2 e) 3

Correct Answer: -1

9. Which sequence of five knots will result in a cubic ($d = 3$) B-spline which has continuous second derivative?

a) $0, 1, 1, 1, 1$ b) $0, 1, 2, 3, 4$ c) $0, 1, 1, 1, 2$ d) $0, 1, 1, 2, 2$ e) $0, 0, 1, 2, 2$

Correct Answer: $0, 1, 2, 3, 4$

10. Which sequence of five knots will result in a cubic ($d = 3$) B-spline which has continuous first, but not second, derivative?

a) $0, 1, 1, 1, 1$ b) $0, 1, 2, 3, 4$ c) $0, 1, 1, 1, 2$ d) $0, 1, 1, 2, 2$ e) $0, 1, 2, 2, 2$

Correct Answer: $0, 1, 1, 2, 2$