Test ID: TESTID Name: ____________________

MAT 300/500 **Practice Quiz** Spring 2021

1. Let $\mathbf{t} = \{-1, -1, 0, 0, 0, 0, 0, 1, 1, 1, 2, 2, 3, 4, 4, 4, 4, 5, 5, 5, 6\}$. How many non-zero B-splines of degree $d = 2$ are attached to this knot sequence?

   a) 13 b) 15 c) 14 d) 11 e) 12

2. Same $\mathbf{t}$ as in the previous question. What is the minimum number of knot values that should be deleted from $\mathbf{t}$ to form a new knot sequence $\mathbf{t}'$ so that all the B-splines associated to $\mathbf{t}'$ are non-zero?

   a) 2 b) 4 c) 5 d) 1 e) 3

3. Same $\mathbf{t}'$ as in the previous question. What is the minimum number of knot values that should be added to the ends of $\mathbf{t}'$ to form a new knot sequence $\mathbf{t}''$ so that the B-splines associated to $\mathbf{t}''$ are a basis of a vector space $V = P^k_{d,\mathbf{r}}[u_0, \ldots, u_k]$? Only add the new knot values on the end, on one or both sides of $\mathbf{t}'$ and do this with the minimal number of knots possible. Hint: check the condition that there are the right number of knots on each end of $\mathbf{t}''$, either $\leq u_0$ or $\geq u_k$.

   a) 5 b) 4 c) 3 d) 1 e) 2

4. Assume that any degree 1 B-spline $\mathcal{B}^1_i(t)$ with knot values $t_i < t_{i+1} < t_{i+2}$ (no repeated knots) is a continuous hat function with value 1 at $t = t_{i+1}$ and value zero at t_i and t_{i+2}. Also assume the derivative formula for B-splines:

$$\frac{d}{dt}\mathcal{B}^d_i(t) = d\left(\frac{\mathcal{B}^{d-1}_i(t)}{t_{i+d} - t_i} - \frac{\mathcal{B}^{d-1}_{i+1}(t)}{t_{i+d+1} - t_{i+1}}\right)$$

   Let $f(t) = \mathcal{B}^2_0(t)$ be the quadratic B-spline with knot sequence $0, 1, 2, 3$. Find $f'(2)$:

   a) $\frac{1}{2}$ b) -1 c) $-\frac{1}{2}$ d) 0 e) 1

5. Let $f(t) = \mathcal{B}^2_0(t)$ be the quadratic B-spline with knot sequence $0, 1, 2, 3$. Find $f'(1.5)$:

   a) $\frac{1}{2}$ b) -1 c) $-\frac{1}{2}$ d) 0 e) 1

6. What is the dimension of the polynomial vector space $P_{7,\{x,y\}}$? (polynomials in x and y with total degree at most 7).

   a) 12 b) 36 c) 24 d) 45 e) 21

7. Let V and W be polynomial vector spaces where $V = P_3$ considered as functions of x on the interval $[0, 3]$, and $W = P_4$ considered as functions of y on the interval $[1, 2]$. Let $T = V \otimes W$, the tensor product space of V and W, and let $U = P_{7,\{x,y\}}$, the space of polynomials of total degree at most 7 in x and y. Choose a function $f(x, y)$ which is in U but not in T:

   a) $x^2(x + y^5)$ b) $x^4(x^3 + y^4)$ c) $x^3(y + y^2)$ d) $x^2(y^2 + y^6)$ e) $x^3(y^4 + y^3)$

8. Suppose a *modified* bicubic surface is defined to have z-values at the corner points of the unit square: $(0, 0)$, $(1, 0)$, $(0, 1)$, and $(1, 1)$, and also at 12 other chosen points. We would like to find a bicubic function $z = f(x, y)$, in the tensor product space $P_{3,x} \otimes P_{3,y}$ which interpolates this input data for any outputs given by some data function g. Which collection of 3 input points would definitely not work as a subset of the 12 inputs? (Hint: use your knowledge of single variable polynomial interpolation.)

   a) $(\frac{1}{2}, 0)$, $(1, \frac{1}{2})$, $(\frac{1}{2}, \frac{1}{2})$ b) $(\frac{1}{4}, 0)$, $(0, \frac{1}{4})$, $(\frac{3}{4}, 0)$ c) $(\frac{1}{4}, 0)$, $(\frac{1}{2}, 0)$, $(\frac{3}{4}, 0)$ d) $(0, \frac{1}{2})$, $(\frac{1}{2}, 0)$, $(\frac{1}{2}, \frac{1}{2})$ e) $(1, \frac{1}{2})$, $(\frac{1}{2}, 1)$, $(\frac{1}{2}, \frac{1}{2})$