

MAT 300/500

Practice Quiz

Spring 2021

1. Let $\mathbf{t} = \{-1, -1, 0, 0, 0, 0, 0, 1, 1, 1, 2, 2, 3, 4, 4, 4, 4, 5, 5, 5, 6\}$. How many non-zero B -splines of degree $d = 2$ are attached to this knot sequence?

a) 13 b) 15 c) 14 d) 11 e) 12

Correct Answer: 15

2. Same \mathbf{t} as in the previous question. What is the minimum number of knot values that should be deleted from \mathbf{t} to form a new knot sequence \mathbf{t}' so that all the B -splines associated to \mathbf{t}' are non-zero?

a) 2 b) 4 c) 5 d) 1 e) 3

Correct Answer: 3

3. Same \mathbf{t}' as in the previous question. What is the minimum number of knot values that should be added to the ends of \mathbf{t}' to form a new knot sequence \mathbf{t}'' so that the B -splines associated to \mathbf{t}'' are a basis of a vector space $V = P_{d,r}^k[u_0, \dots, u_k]$? Only add the new knot values on the end, on one or both sides of \mathbf{t}' and do this with the minimal number of knots possible. Hint: check the condition that there are the right number of knots on each end of \mathbf{t}'' , either $\leq u_0$ or $\geq u_k$.

a) 5 b) 4 c) 3 d) 1 e) 2

Correct Answer: 3

4. Assume that any degree 1 B -spline $\mathcal{B}_i^1(t)$ with knot values $t_i < t_{i+1} < t_{i+2}$ (no repeated knots) is a continuous hat function with value 1 at $t = t_{i+1}$ and value zero at t_i and t_{i+2} . Also assume the derivative formula for B -splines:

$$\frac{d}{dt}\mathcal{B}_i^d(t) = d \left(\frac{\mathcal{B}_i^{d-1}(t)}{t_{i+d} - t_i} - \frac{\mathcal{B}_{i+1}^{d-1}(t)}{t_{i+d+1} - t_{i+1}} \right)$$

Let $f(t) = \mathcal{B}_0^2(t)$ be the quadratic B -spline with knot sequence 0, 1, 2, 3. Find $f'(2)$:

a) $\frac{1}{2}$ b) -1 c) $-\frac{1}{2}$ d) 0 e) 1

Correct Answer: -1

5. Let $f(t) = \mathcal{B}_0^2(t)$ be the quadratic B -spline with knot sequence 0, 1, 2, 3. Find $f'(1.5)$:

a) $\frac{1}{2}$ b) -1 c) $-\frac{1}{2}$ d) 0 e) 1

Correct Answer: 0

6. What is the dimension of the polynomial vector space $P_{7,\{x,y\}}$? (polynomials in x and y with total degree at most 7).

a) 12 b) 36 c) 24 d) 45 e) 21

Correct Answer: 36

7. Let V and W be polynomial vector spaces where $V = P_3$ considered as functions of x on the interval $[0, 3]$, and $W = P_4$ considered as functions of y on the interval $[1, 2]$. Let $T = V \otimes W$, the tensor product space of V and W , and let $U = P_{7,\{x,y\}}$, the space of polynomials of total degree at most 7 in x and y . Choose a function $f(x, y)$ which is in U but not in T :

a) $x^2(x + y^5)$ b) $x^4(x^3 + y^4)$ c) $x^3(y + y^2)$ d) $x^2(y^2 + y^6)$ e) $x^3(y^4 + y^3)$

Correct Answer: $x^2(x + y^5)$

8. Suppose a *modified* bicubic surface is defined to have z -values at the corner points of the unit square: $(0, 0)$, $(1, 0)$, $(0, 1)$, and $(1, 1)$, and also at 12 other chosen points. We would like to find a bicubic function $z = f(x, y)$, in the tensor product space $P_{3,x} \otimes P_{3,y}$ which interpolates this input data for any outputs given by some data function g . Which collection of 3 input points would definitely not work as a subset of the 12 inputs? (Hint: use your knowledge of single variable polynomial interpolation.)

- a) $(\frac{1}{2}, 0)$, $(1, \frac{1}{2})$, $(\frac{1}{2}, \frac{1}{2})$ b) $(\frac{1}{4}, 0)$, $(0, \frac{1}{4})$, $(\frac{3}{4}, 0)$ c) $(\frac{1}{4}, 0)$, $(\frac{1}{2}, 0)$, $(\frac{3}{4}, 0)$ d) $(0, \frac{1}{2})$, $(\frac{1}{2}, 0)$, $(\frac{1}{2}, \frac{1}{2})$
 e) $(1, \frac{1}{2})$, $(\frac{1}{2}, 1)$, $(\frac{1}{2}, \frac{1}{2})$

Correct Answer: $(\frac{1}{4}, 0)$, $(\frac{1}{2}, 0)$, $(\frac{3}{4}, 0)$