

MAT 300/500 Homework 1

Spring 2025

Due date: Friday, January 17

Note: It is recommended that you use a symbolic math package such as PARI or SAGE, or a calculator that does linear algebra, to work some of these problems. Be sure to check ALL your work. This is simple to do with such a package. Wrong answers will receive zero!

- Let $p(t) = 2 + t - 3t^2$. Find the coordinate vector of $p(t)$ in each of the following bases. (The order from top to bottom of the coordinates in the coordinate vector corresponds to the order from left to right in the basis.)
 - The shifted basis $\{1, t - 2, (t - 2)^2\}$
 - The Van der Monde basis $V(1, 2, 3) = \{(t - 1)^2, (t - 2)^2, (t - 3)^2\}$
 - The top-down basis $\{(t - 1)^2, (t - 3)^2, t - 3\}$
 - The Bernstein basis $B(2) = \{B_0^2(t), B_1^2(t), B_2^2(t)\}$.
- Find a cubic polynomial in standard basis form ($y = a_0 + a_1t + a_2t^2 + a_3t^3$) which passes through the points $(0, 3)$, $(1, 3)$, $(-1, 7)$, and $(2, 1)$. (Use a 4×4 linear system with the coefficients of the cubic as the variables, and solve.)
 - Find the same polynomial in the shifted basis $\{1, t - 2, (t - 2)^2, (t - 2)^3\}$.
 - Find the same polynomial in the Bernstein basis $B(3) = \{B_0^3(t), B_1^3(t), B_2^3(t), B_3^3(t)\}$.
 - Find the same polynomial in the Van der Monde basis $V(1, 2, 3, 4)\{(t - 1)^3, (t - 2)^3, (t - 3)^3, (t - 4)^3\}$.
- Find the equation of a cubic polynomial which passes through $(1, -2)$, with slope -2 at this point, and through $(-1, 2)$, with slope 2 at this point. (Use a linear system approach, with the polynomial and its derivative.)
 - Find the same polynomial in the shifted basis $\{1, t - 2, (t - 2)^2, (t - 2)^3\}$.
 - Find the same polynomial in the Bernstein basis $B(3)$.
 - Find the same polynomial in the Vandermonde basis $V(1, 2, 3, 4)$.
- For each of the following sets of polynomials, determine if the set is top-down or not, and whether the set is linearly independent or not in P_2 .
 - $\{t^2, (t - 3)^2, t - 2\}$
 - $\{(t - 2)^2, (t - 1)^2, t - 2\}$
 - $\{(t - 2)^2, (t - 1)^2, t - \frac{3}{2}\}$
 - $\{(t - 3)^2, t - 2, 4\}$