MAT 300/500 Homework 1
Spring 2021

Due date: Tuesday, January 19

Note: It is recommended that you use a symbolic math package such as PARI or SAGE, or a calculator that does linear algebra, to work some of these problems. Be sure to check ALL your work. This is simple to do with such a package. Wrong answers will receive zero!

1. Let $p(t) = 2 + t - 3t^2$. Find the coordinate vector of $p(t)$ in each of the following bases. (The order from top to bottom of the coordinates in the coordinate vector corresponds to the order from left to right in the basis.)
   (a) The shifted basis $\{1, t-2, (t-2)^2\}$
   (b) The Van der Monde basis $V(1,2,3) = \{(t-1)^2, (t-2)^2, (t-3)^2\}$
   (c) The top-down basis $\{(t-1)^2, (t-3)^2, t-3\}$
   (d) The Bernstein basis $B(2) = \{B_0^2(t), B_1^2(t), B_2^2(t)\}$.
2. (a) Find a cubic polynomial in standard basis form ($y = a_0 + a_1 t + a_2 t^2 + a_3 t^3$) which passes through the points $(0,3)$, $(1,3)$, $(-1,7)$, and $(2,1)$. (Use a 4×4 linear system with the coefficients of the cubic as the variables, and solve.)
   (b) Find the same polynomial in the shifted basis $\{1, t-2, (t-2)^2, (t-2)^3\}$.
   (c) Find the same polynomial in the Bernstein basis $B(3) = \{B_0^3(t), B_1^3(t), B_2^3(t), B_3^3(t)\}$.
   (d) Find the same polynomial in the Van der Monde basis $V(1,2,3,4)\{(t-1)^3, (t-2)^3, (t-3)^3, (t-4)^3\}$.
3. (a) Find the equation of a cubic polynomial which passes through $(1,-2)$, with slope -2 at this point, and through $(-1,2)$, with slope 2 at this point. (Use a linear system approach, with the polynomial and its derivative.)
   (b) Find the same polynomial in the shifted basis $\{1, t-2, (t-2)^2, (t-2)^3\}$.
   (c) Find the same polynomial in the Bernstein basis $B(3)$.
   (d) Find the same polynomial in the Vandermonde basis $V(1,2,3,4)$.
4. For each of the following sets of polynomials, determine if the set is top-down or not, and whether the set is linearly independent or not in P_2.
   (a) $\{t^2, (t-3)^2, t-2\}$
   (b) $\{(t-2)^2, (t-1)^2, t-2\}$
   (c) $\{(t-2)^2, (t-1)^2, t-\frac{3}{2}\}$
   (d) $\{(t-3)^2, t-2, 4\}$