MAT 300/500 Homework 2

Spring 2019

Due Date: Thursday January 31, 2019

1. Derive the formula for the derivative of the Bernstein polynomials without using induction or recursion, just the definitions.

$$\frac{d}{dt}B_i^d(t) = d(B_{i-1}^{d-1}(t) - B_i^{d-1}(t))$$

(In class we use induction and the recursion property. You do it with direct manipulation of binomial coefficients and indices.)

2. Show that the quadratic Cumulative Bernstein polynomials are a basis of P_2 without reference to the standard basis.

3. Show that the derivative of the Cumulative Bernstein polynomials is: $\frac{d}{dt}C_i^d(t) = dB_{i-1}^{d-1}(t) \quad i = 0, \ldots, d.$

(Hint: Use the derivative property for $B_i^d(t)$ to form a sum with lots of cancellation.)

4. Find the values of the Vandermonde or Confluent Vandermonde determinants:

(a) $\begin{vmatrix} 1 & 2 & 2^2 \\ 1 & 3 & 3^2 \\ 1 & 5 & 5^2 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 4 & 4^2 & 4^3 & 4^4 \\ 1 & 5 & 5^2 & 5^3 & 5^4 \\ 1 & 6 & 6^2 & 6^3 & 6^4 \\ 1 & 7 & 7^2 & 7^3 & 7^4 \\ 1 & 9 & 9^2 & 9^3 & 9^4 \end{vmatrix}$ (c) $\begin{vmatrix} 1 & 4 & 4^2 & 4^3 & 4^4 \\ 0 & 1 & 2*4 & 3*4^2 & 4*4^3 \\ 0 & 0 & 2 & 6*4 & 12*4^2 \\ 0 & 0 & 0 & 6 & 24*4 \\ 1 & 3 & 3^2 & 3^3 & 3^4 \end{vmatrix}$

5. Write down a formula for a polynomial $P(x)$ using a van der Monde determinant (with powers of x in one row or column, as done in class), such that $P(x)$ has roots at $x = 1$, 3, 4, and 6, and $P(2) = 2^5 \cdot 3^2 \cdot 5$. Find $P(5)$.