1. Let \( g(t) = \cos(\pi t) \). In each part use the Newton form to find the quadratic interpolating (or osculating) polynomial \( p(t) \) which agrees with the function \( g(t) \) for each of the sequences \([t_0, t_1, t_2]\): Sketch three graphs, one for each polynomial together with the function \( g(t) \).
   i) \([0, 1, 2]\) ii) \([0, 0, 2]\) iii) \([-1, -1, 2]\)

2. Let \( p(t) \) be the interpolating polynomial with data values \([t_0, t_1, t_2, t_3]\) and let \( p_0(t) \) be the interpolating polynomial with data values \([t_0, t_1, t_2]\) and let \( p_1(t) \) be the interpolating polynomial with data values \([t_1, t_2, t_3]\). Assume the data function \( g(t) \) has values: \( g(0) = 2, g(1) = 3, g(2) = 4, \) and \( g(3) = -1 \). Find the Newton forms for \( p_0(t) \) and \( p_1(t) \) and show that the formula
   \[
   p(t) = \frac{t - t_0}{t_d - t_0} p_1(t) + \frac{t_d - t}{t_d - t_0} p_0(t)
   \]
   works for \( d = 3 \).

3. Find the product of polynomial and piecewise polynomial as a linear combination of truncated power functions \((t - c)^k\):
   \[
   (t - 1)^3(t^2 - 3t + 5).
   \]

4. Write the piecewise polynomial function \( f(t) \) as a linear combination of truncated power functions \((t - c)^k\):
   \[
   f(t) = \begin{cases} 
   0, & 0 \leq t < 1 \\
   t - 1, & 1 \leq t < 2 \\
   0, & 2 \leq t \leq 3
   \end{cases}
   \]

5. Find the osculating polynomial \( p(t) \) for the data values \([0, 0, 1, 1]\) with \( g(0) = 2, g'(0) = -3, g(1) = -5, \) and \( g'(1) = -11 \). Recompute the polynomial \( p(t) \) with the sequence \([1, 1, 0, 0]\). Write both answers in standard basis to check that they are the same.

6. Find a polynomial \( p(t) \) of degree 6 which has a zero of multiplicity 2 at \( t = 1 \) and a zero of multiplicity 3 at \( t = 2 \), and also satisfies: \( p(0) = 2 \) and \( p'(0) = 1 \). What is the other root of \( p(t) \)? Do this problem in the following three different ways. Check that the answer in each case is the same by putting in standard form.
   (a) Write \( p(t) \) in factored form with an undetermined coefficient \( a \) and an undetermined root \( b \). Solve for \( a \) and \( b \) using the conditions on \( p(0) \) and \( p'(0) \).
   (b) Use a divided difference table for the osculating polynomial. Find \( a \) and \( b \) from the Newton form. (Hint: It is easiest if you arrange the values \( t_i \) to have lots of zeros along the top of the triangle.)
   (c) Write a linear system involving coefficients for the standard basis as the variables, using the derivative definition of multiplicity. Solve it with a symbolic algebra tool.