MAT 300/500 Homework 3 – Spring 2021

Due Date: Tuesday, February 9

1. Let $g(t) = \cos(\frac{\pi}{2}t)$. In each part use the Newton form to find the quadratic interpolating (or osculating) polynomial $p(t)$ which agrees with the function $g(t)$ for each of the sequences $[t_0, t_1, t_2]$: Sketch three graphs, one for each polynomial together with the function $g(t)$.

   i) $[0, 1, 2]$ ii) $[0, 0, 2]$ iii) $[-1, 2, 2]$

2. Let $p(t)$ be the interpolating polynomial with data values $[t_0, t_1, t_2, t_3] = [0, 1, 2, 3]$ and let $p_0(t)$ be the interpolating polynomial with data values $[t_0, t_1, t_2] = [0, 1, 2]$ and let $p_1(t)$ be the interpolating polynomial with data values $[t_1, t_2, t_3] = [1, 2, 3]$. Assume the data function $g(t)$ has values: $g(0) = 2$, $g(1) = 3$, $g(2) = 4$, and $g(3) = -1$. Find the Newton forms for $p_0(t)$ and $p_1(t)$ and show that the formula

$$p(t) = \frac{t - t_0}{t_d - t_0} p_1(t) + \frac{t_d - t}{t_d - t_0} p_0(t)$$

   works for $d = 3$.

3. Find the product of polynomial and piecewise polynomial as a linear combination of truncated power functions $(t - c)_+^k$:

$$(t-1)_+^3(t^2 - 3t + 5).$$

4. Write the piecewise polynomial function $f(t)$ as a linear combination of truncated power functions $(t - c)_+^k$:

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t - 1, & 1 \le t < 2 \\ 0, & 2 \le t \le 3 \end{cases}$$

5. Find the osculating polynomial $p(t)$ for the data values $[0, 0, 1, 1]$ with $g(0) = 2$, $g'(0) = -3$, $g(1) = -5$, and $g'(1) = -11$. Recompute the polynomial $p(t)$ with the sequence $[1, 1, 0, 0]$. Write both answers in standard basis to check that they are the same.

6. Find a polynomial $p(t)$ of degree 6 which has a zero of multiplicity 2 at $t = 1$ and a zero of multiplicity 3 at $t = 2$, and also satisfies: $p(0) = 2$ and $p'(0) = 1$. What is the other root of $p(t)$? Do this problem in the following three different ways. Check that the answer in each case is the same by putting in standard form.

   (a) Write $p(t)$ in factored form with an undetermined coefficient a and an undetermined root b. Solve for a and b using the conditions on $p(0)$ and $p'(0)$.

   (b) Use a divided difference table for the osculating polynomial. Find a and b from the Newton form. (Hint: It is easiest if you arrange the values t_i to have lots of zeros along the top of the triangle.)

   (c) Write a linear system involving coefficients for the standard basis as the variables, using the derivative definition of multiplicity. Solve it with a symbolic algebra tool.