# MAT 300/500 Homework 4
# Spring 2019

Due Date: Monday, March 4

1. Find the implicit form of the Bezier curve with control points $P_0 = (1,2)$, $P_1 = (0,0)$, and $P_2 = (2,0)$, using the tangent construction. Check the discriminant $\Delta = B^2 - 4AC$ to see that it is zero.

2. Find the polar forms in 2 variables $u_1, u_2$, and then find the control points, of the following parametric polynomial functions: (following the examples done in class)

   (a) $\gamma(t) = (2 - t, 1 - 2t + 4t^2)$

   (b) $\gamma(t) = (2 + t^2, 1 - 3t)$

3. (a) Let $P_0 = (-1,4)$ and $P_2 = (1,0)$. Find a third point $P_1$ so that the Bezier curve $\gamma_{[P_0,P_1,P_2]}(t) = \gamma(t)$ lies on the parabola $y = (x-1)^2$.

   (b) Let $\alpha(t) = \gamma(3t-1)$. Expand and find the quadratic parametric form for $\alpha(t)$ (in terms of the standard basis for the vector space $P_2$.)

   (c) Convert the polynomials in the previous part to the Bernstein basis and obtain the BB form for $\alpha(t)$, and hence also the control points for $\alpha(t)$.

   (d) Call the control points for $\alpha(t)$ $Q_0$, $Q_1$, and $Q_2$. Find the tangent lines to the parabola at $Q_0$ and $Q_2$ and verify that they intersect at $Q_1$.

   (e) Find the polar form for $\gamma(t)$ using the Bernstein basis.

   (f) Compute the control points for $\alpha(t)$ $Q_0$, $Q_1$, and $Q_2$, by using the polar form for $\gamma(t)$, and verify that they are the same as in the previous parts.

4. (a) Let $P_0 = (-2,0)$, $P_1 = (-1,3)$, and $P_3 = (3,1)$. Find a fourth point $P_2$ so that the Bezier curve $\gamma_{[P_0,P_1,P_2,P_3]}(t) = \gamma(t)$ has $\gamma(\frac{1}{2}) = (\frac{1}{2}, 2)$.

   (b) Let $\alpha(t) = \gamma(3t-1)$. Expand and find the cubic parametric form for $\alpha(t)$ (in terms of the standard basis for $P_3$.)

   (c) Convert the polynomials in the previous part to the Bernstein basis and obtain the BB form for $\alpha(t)$, and hence also the control points for $\alpha(t)$.

   (d) Call the control points for $\alpha(t)$ $Q_0$, $Q_1$, $Q_2$, and $Q_3$. Find the tangent lines to $\gamma(t)$ in parametric form at $Q_0$ and $Q_3$ and verify that the other control points each lie on one of these lines.

   (e) Find the polar form for $\gamma(t)$ using the Bernstein basis.

   (f) Compute the control points for $\alpha(t)$ $Q_0$, $Q_1$, and $Q_2$, by using the polar form for $\gamma(t)$, and verify that they are the same as in the previous parts.

5. Let $\gamma(t)$ be the quadratic Bezier curve with control points $P_0 = (1,0)$ and $P_1 = (2,2)$, and implicit form $y = x^2 - 1$. Find a third control point $P_2$ for this curve, using the tangent line properties.