

MAT 300/500 Homework 5

Spring 2019

Due Date: Thursday, March 14

1. Let V be the vector space $P_{1,0}^4([0, 1, 2, 3, 4])$ of continuous linear splines on the indicated sequence of intervals. Let f be the spline in V whose graph is the piecewise linear path connecting the points: $(0, 1)$, $(2, -1)$, $(3, 1)$, and $(4, 1)$ (and consisting of exactly one line between each pair of points.) Let

$$F = \{1, t, (t-1)_+, (t-2)_+, (t-3)_+\}$$

(as functions restricted to $[0, 4]$) be a basis of V .

Let h be the ‘hat function’:

$$h(t) = \begin{cases} 0, & t < 0 \\ t, & 0 \leq t < 1 \\ 2-t, & 1 \leq t < 2 \\ 0, & t \geq 2 \end{cases},$$

and let H be the set of functions

$$\{h_1(t) = h(t+1), h_2(t) = h(t), h_3(t) = h(t-1), h_4(t) = h(t-2), h_5(t) = h(t-3)\}$$

restricted to the interval $[0, 4]$. Then H is also a basis of V .

- (a) Find the coordinate vector of f with respect to the basis F .
 - (b) Find the change of basis matrices from H to F and F to H .
 - (c) Find the coordinate vector of f with respect to the basis H .
2. Show that $P_{a,0}^2([a, c, b])$ is a vector subspace of $P_a^2([a, c, b])$. (Quote a criterion from Linear Algebra to check whether a subset is a subspace. Then use a fact from Calculus to verify that you have a subspace.)
 3. Considered as subspaces of $C^0([0, 3])$, the space of all continuous functions on the interval $[0, 3]$, what is the intersection of the vector spaces $P_{3,0}^2([0, 1, 3])$ and $P_{3,0}^2([0, 2, 3])$? Give reasons for your answer, without reference to any bases.