MAT 300/500 Homework 5 – Spring 2024

Due Date: Friday, March 8

1. Let $g(t) = 1/t$ and $h(t) = 1/t^2$ and $f(t) = g(t)h(t) = 1/t^3$. Find the divided differences $[-1, 1, 2]f$, $[-1, 1, 2]g$ and $[-1, 1, 2]h$ by computing the divided difference table in each case. Then verify Leibniz's Formula for computing $[-1, 1, 2]f$ with the summation of products from g and h.

2. Define, for real constant c and positive integer k:
$$(t-c)_+^k = \begin{cases} 0, & t < c \\ (t-c)^k, & t \geq c \end{cases} \quad \text{and} \quad (t-c)_+^0 = \begin{cases} 0, & t < c \\ 1, & t \geq c \end{cases}$$
Let $g(t) = t - 2$ and $h(t) = (t-2)_+^3$ and $f(t) = g(t)h(t) = (t-2)_+^4$. Find the divided differences $[1, 3, 4]f$, $[1, 3, 4]g$ and $[1, 3, 4]h$ by computing the divided difference table in each case. Then verify Leibniz's Formula for computing $[1, 3, 4]f$ with the summation of products from g and h.

3. Find the implicit form of the Bezier curve with control points $P_0 = (1, 2)$, $P_1 = (0, 0)$, and $P_2 = (2, 0)$, using the tangent construction. Verify with the discriminant that this curve is a parabola.

4. Find the polar forms in d variables $u_1, \ldots, u_d$, of the following polynomials:
   (a) t^4, $d = 5$
   (b) $B_3^4(t)$, $d = 4$

5. Find the polar forms in d variables $u_1, \ldots, u_d$, and then find the control points, of the following parametric polynomial functions:
   (a) $\gamma(t) = (2 - t, 1 - 2t + 4t^2)$, $d = 2$
   (b) $\gamma(t) = (2 + t - t^3, 1 - t + 3t^2)$, $d = 3$

6. (a) Let $P_0 = (-1, 4)$ and $P_2 = (1, 0)$. Find a third point P_1 so that the Bezier curve $\gamma_{[P_0,P_1,P_2]}(t) = \gamma(t)$ lies on the parabola $y = (x-1)^2$.
   (b) Let $\alpha(t) = \gamma(3t - 1)$. Expand and find the quadratic parametric form for $\alpha(t)$ (in terms of the standard basis for the vector space P_2.)
   (c) Convert the polynomials in the previous part to the Bernstein basis and obtain the BB form for $\alpha(t)$, and hence also the control points for $\alpha(t)$.
   (d) Call the control points for $\alpha(t)$ Q_0, Q_1, and Q_2. Find the tangent lines to the parabola at Q_0 and Q_2 and verify that they intersect at Q_1.
   (e) Find the polar form $F[u_1, u_2]$ for $\gamma(t)$.
   (f) Use the polar form to find both sets of control points for $\gamma(t)$ and $\alpha(t)$.
   (g) Let $\beta(t) = \alpha(2t + 3)$. Find a and b so that $\beta(t) = \gamma((1-t)a + tb)$.
   (h) Use the polar form to find the control points of $\beta(t)$.