Test ID: TESTID Name: _____

MAT 320 **Quiz 3** Spring 2024

1. Consider the digital filter F_1: $y_t = x_t - \frac{1}{4}x_{t-1}$. What is the transfer function $\mathcal{H}(z)$ for this filter?
   a) $1 - \frac{1}{4}z$ b) $\frac{z}{z-\frac{1}{4}}$ c) $\frac{1}{1-\frac{1}{4}z}$ d) $1 - \frac{1}{4}z^{-1}$ e) $1 - 4z$
   Correct Answer: $1 - \frac{1}{4}z^{-1}$

2. Same filter F_1 as in the previous question. This filter has a zero at z equal to:
   a) $-\frac{1}{4}$ b) $\frac{1}{4}$ c) $\frac{1}{4}(1+i)$ d) $-\frac{1}{4}(1+i)$ e) 0
   Correct Answer: $\frac{1}{4}$

3. Same filter F_1 as in the previous question. This filter has a pole at z equal to:
   a) $-\frac{1}{4}$ b) $\frac{1}{4}$ c) $\frac{1}{4}(1+i)$ d) $-\frac{1}{4}(1+i)$ e) 0
   Correct Answer: 0

4. Same filter F_1 as in the previous question. What is the frequency response $H(\omega)$ of this filter for $\omega = 0$?
   a) $\frac{3}{4}$ b) $\frac{4}{3}$ c) 1 d) $\frac{\sqrt{3}}{5}(1+i)$ e) $\frac{\sqrt{2}}{5}(1+i)$
   Correct Answer: $\frac{3}{4}$

5. Same filter F_1 as in the previous question. What is the magnitude response $|H(\omega)|$ of this filter for $\omega = \pi/2$?
   a) $\sqrt{\frac{17}{18}}$ b) $\sqrt{\frac{12}{17}}$ c) $\frac{\sqrt{17}}{4}$ d) $\frac{\sqrt{17}}{5}$ e) $\frac{\sqrt{5}}{17}$
   Correct Answer: $\frac{\sqrt{17}}{4}$

6. Same filter F_1 as in the previous question. If the input $\mathbf{x}$ is the unit impulse signal: $(1, 0, 0, 0, \ldots)$ then what is the output value y_2? (Assume values with index less than zero are equal to 0.)
   a) $\frac{1}{4}$ b) $-\frac{1}{4}$ c) 1 d) 2 e) 0
   Correct Answer: 0

7. Suppose we use sampling rate $f_s = 44100$ Hz. What is the frequency of smallest positive value which is an alias of the frequency 32100 Hz, but is not equal to this frequency?
   a) 12050 b) 12000 c) 12500 d) 10050 e) 6000
   Correct Answer: 12000

8. Let $\{\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2\}$ be the Fourier basis of $\mathbb{C}^3$, where $\mathbf{u}_k$ is the vector obtained by sampling the phasor $e^{i\frac{2\pi}{3}kt}$ at the t-values $0, 1, 2$. What is the sum $\mathbf{u}_1 + \mathbf{u}_2$?
   a) $\begin{pmatrix} 3 \\ e^{-i\frac{2\pi}{3}} \\ e^{-i\frac{4\pi}{3}} \end{pmatrix}$ b) $\begin{pmatrix} 2 \\ e^{i\frac{\pi}{3}} \\ e^{i\frac{2\pi}{3}} \end{pmatrix}$ c) $\begin{pmatrix} 2i \\ i \\ i^2 \end{pmatrix}$ d) $\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ e) $\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$
   Correct Answer: $\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$

9. Let $\{\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2\}$ be the Fourier basis of $\mathbb{C}^3$, where $\mathbf{u}_k$ is the vector obtained by sampling the phasor $e^{i\frac{2\pi}{3}kt}$ at the t-values $0, 1, 2$. What is the complex inner product $\mathbf{u}_1 \bullet \mathbf{u}_2$?
   a) 0 b) i c) 2 d) -2 e) 1
   Correct Answer: 0

10. Let $\mathbf{x}$ be a complex vector with coordinates $1, 0, 1$. Find $X_1 = DFT(\mathbf{x}, 3, 1)$.
    a) i b) 1 c) 2 d) $e^{i\frac{\pi}{3}}$ e) $e^{i\frac{2\pi}{3}}$
    Correct Answer: $e^{i\frac{\pi}{3}}$