Test ID: TESTID Name: _____

MAT 320 **Quiz 6** Fall 2022

Some properties of functions which may or may not hold for a given function f:

- *computable*: f can be evaluated, which means that for any real number x as input to f there exists a real number $f(x)$ as output of f. (Also can be stated as: f has domain all real numbers.)
- *differentiable*: f can be differentiated, which means that for any real number x, there exists a slope $f'(x)$ which is the derivative of f at x. Equivalently, $f'(a)$ is the limit as x approaches a of the difference quotient given by slopes of secant lines through the points $(x, f(x))$ and $(a, f(a))$.
- *integrable*: f can be integrated, which means that for any two real numbers a and b, the definite integral $\int_a^b f(x)\,dx$ exists and equals a real number. The integral can be realized as the area contained between the graph of f and the x axis when $f(x)$ is positive, and the negative area when $f(x)$ is negative.
- *continuous*: f is continuous means that there are no breaks or jumps in the graph of f, or equivalently that the limit as x approaches any constant a of the values $f(x)$ exists and equals the function value $f(a)$.

1. Which of the properties below are NOT TRUE for the square wave function, which is defined on one full period as 1 on $[0, T/2)$ and -1 on $[T/2, T)$. Note: each property implies that it holds true for all real number inputs, except for integrable, which means that the integral of the function from a to b exists for any real numbers a and b.

   i) continuous ii) differentiable iii) integrable iv) computable

   a) ii) and iii) only b) i) and ii) only c) ii) only d) iii) only e) iv) only

   Correct Answer: i) and ii) only

2. Same question for the triangle wave function, which can be obtained as the integral of the square wave.

   a) ii) and iii) only b) i) and ii) only c) ii) only d) iii) only e) iv) only

   Correct Answer: ii) only

3. Let $f(t)$ be the square wave with period $T = 2$ defined on its first period as 1 on the interval $[0, 1)$ and as -1 on the interval $[1, 2)$. Find the inner product $< f(t), e^{i\pi t} >$

   a) $\frac{2}{\pi}$ b) $\frac{-2i}{\pi}$ c) $\frac{i}{2\pi}$ d) 1 e) 0

   Correct Answer: $\frac{-2i}{\pi}$

4. Same $f(t)$ as in the previous question. What is the inner product $< f(t), e^{i2\pi t} >$?

   a) $\frac{-2i}{\pi}$ b) $\frac{2}{\pi}$ c) $\frac{2i}{\pi}$ d) 1 e) 0

   Correct Answer: 0

5. Let $g(t)$ be the triangle wave function made up of the lines $y = t - \frac{1}{4}$ on the interval $[0, \frac{1}{2})$ and $y = \frac{3}{4} - t$ on the interval $[\frac{1}{2}, 1)$, extended to all values of t by $g(t + k) = g(t)$, for any integer k. Write the Fourier series for $g(t)$ as a real sum of cosine functions. What is the coefficient of $\cos 6\pi t$?

a) $-\frac{1}{9}\pi^2$ b) $-\frac{\pi}{3}$ c) $-\frac{2}{9\pi^2}$ d) $\frac{3\pi}{2}$ e) $\frac{9}{2\pi^2}$

Correct Answer: $-\frac{2}{9\pi^2}$

6. Same function $g(t)$ as in the previous question. Let $b(t)$ be the buzz function defined as: $\sum_{k=-\infty}^{+\infty} \delta(t-k)$ where $\delta(t)$ is the 'delta function' which is zero except for $t = 0$ where it has a concentrated area of value one. What is the value of the integral: $\int_{1/2}^{3/2} g(t)b(t)\ dt$?

a) $-\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $-\frac{1}{2}$ e) 0

Correct Answer: $-\frac{1}{4}$

7. Let $f(t)$ be the modified square wave with period $T = 2$ defined on its first period as 1 on the interval $[0, 1)$ and as -1 on the interval $[1, \frac{3}{2})$, and zero on the interval $[\frac{3}{2}, 2)$. Find the value of the integral:

$$\int_0^2 f(t)\ dt$$

a) $\frac{2}{\pi}$ b) $\frac{1}{2}$ c) $\frac{1}{\pi}$ d) 1 e) 2

Correct Answer: $\frac{1}{2}$

8. Same $f(t)$ as in the previous question. What is the inner product $< f(t), e^{i2\pi t} >$?

a) $\frac{-2i}{\pi}$ b) $\frac{2}{\pi}$ c) $\frac{i}{2\pi}$ d) 1 e) 0

Correct Answer: $\frac{i}{2\pi}$

9. Same function $f(t)$ as in the previous question. If f is written as a Fourier series, what is the value of the coefficient c_1?

$$f(t) = \sum_{k=-\infty}^{+\infty} c_k e^{ik\omega_0 t}.$$

a) $\frac{2i}{\pi}$ b) $\frac{i}{2\pi}$ c) $\frac{1-i}{\pi}$ d) 1 e) 0

No Correct Answer Given (all answers receive credit) Correct Answer: $\frac{1}{2\pi}(1 - 3i)$

10. Let $f_a(t)$ be defined as $\frac{3}{a^3}(a^2 - t^2)$ for $-a \le t \le a$, and zero elsewhere. If δ is the Dirac delta distribution (which is a generalized function with area 1 concentrated at $t = 0$), what multiple of δ is defined in the limit: $\lim_{a\to 0} f_a(t)$? (Hint: find the area under the curve $y = f_a(t)$ from $-a$ to a.)

a) 2δ b) 3δ c) 5δ d) 4δ e) δ

Correct Answer: 4δ