MAT 320 Homework 2
Fall 2024

Due date: Wednesday, Sep 18

1. Determine if the following functions are periodic or non-periodic. If the function is periodic, give the smallest possible period. (Note: each function has domain all real numbers, and codomain the complex plane $\mathbb{C}$. This means that the range must be a subset of $\mathbb{C}$.) In each case, also describe the range of f. A description can use any combination of: words, formulas, geometry, and pictures.

   (a) $f(t) = t + t^2 i$
   (b) $f(t) = e^{i\frac{\pi}{3}t}$
   (c) $f(t) = 3e^{i\frac{\pi}{4}t}$
   (d) $f(t) = 3e^{i(\frac{\pi}{4}t+4)}$
   (e) $f(t) = i\sin(\frac{\pi}{3}t)$
   (f) $f(t) = te^{i\frac{\pi}{3}t}$
   (g) $f(t) = e^{t+i\frac{\pi}{3}t}$
   (h) $f(t) = e^{i\frac{\pi}{2}\sin(\frac{\pi}{3}t)}$
   (i) $f(t) = \sin(\frac{\pi}{3}t)e^{i\frac{\pi}{3}t}$

2. Find the complex number z_0 in polar form which has the property that multiplication by z_0 gives a function which rotates all complex numbers by the angle $\pi/6$ counterclockwise and also scales them by 6. Call this function f_{z_0}, so that for any complex number z the function gives $f_{z_0}(z) = z_0 z$. Find the cartesian form of z_0. Find the $2D$ matrix which performs the same operation on points of the plane $\mathbb{R}^2$ as f_{z_0} performs on the complex plane.

3. Show that complex numbers w and z are linearly dependent (as real vectors) if and only if $w\bar{z} \in \mathbb{R}$. (Note: the linear dependence statement uses only the vector space properties of $\mathbb{C}$, but the criterion in this case uses the multiplication of $\mathbb{C}$. To show the if and only if statement is true, you need to prove two implications. For example: "A if and only if B" is true if A implies B and B implies A. So, first assume A and show that B is true, then second assume B and show that A is true.)