MAT 320 Homework 3
Fall 2023

Due date: Wednesday, September 27

You can use SciLab (or other math software) for any part of this homework.

1. Suppose a signal is sampled at the rate $f_s = 44,100$. In each part find the smallest positive frequency which is an alias of, but not equal to, the given frequency:
   (a) 23000 Hz
   (b) 45000 Hz
   (c) 1000 Hz
   (d) 96000 Hz

2. Find the signal to noise ratio SNR for a signal which is sampled with 10 bit values ($B = 10$), assuming that the error is uniformly distributed between 0 and 1/2. Now suppose a signal has values: -340, 223.45, 190.6, -48.2, which are in the range between -2^9 and $2^9 - 1$. Find the RMS value for these four samples (take the average of the squares of the quantization errors, then take the square root).

3. Use a phasor sum to show that

$$2\cos(20\pi t + \pi/3) + 3\cos(20\pi t + \pi/4)$$

   can be written as: $A\cos(20\pi t + \phi)$. Find the constants A and ϕ as decimal approximations. Use your project or a calculator to compute the answers and any conversions from Cartesian to polar form. Show all work for each step.

4. Let $B_4 = \{\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be the Fourier basis for dimension 4, where $\mathbf{u}_k$ is the sampled phasor

$$e^{i\frac{2\pi}{4}kt}, \quad t = 0, 1, 2, 3.$$

   Let A be the matrix of column vectors

$$A = \left(\mathbf{u}_0 \quad \mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{u}_3\right).$$

   (a) Find the determinant of A.
   (b) Verify that the determinant of A equals the product of backward differences from the second column, ie. the product

$$\prod_{0 \le i < j \le 3} z_j - z_i$$

   where

$$\mathbf{u}_1 = \begin{pmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \end{pmatrix}.$$

   Note: the product notation is like a sum, where we use capital sigma, but now here we use capital pi for product. The product is of all factors $z_j - z_i$, where the indexing is over all pairs i, j with $i < j$ and i and j in the range from 0 to 3.
   (c) Find the inverse matrix A^{-1}.

(d) Solve for the coefficients a_0, a_1, a_2, a_3 in the vector equation:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = a_0\mathbf{u}_0 + a_1\mathbf{u}_1 + a_2\mathbf{u}_2 + a_3\mathbf{u}_3$$

using the inverse matrix.

(e) Solve for the coefficients a_0, a_1, a_2, a_3 using dot products. (Note: for the complex dot product

$$(c\mathbf{u}) \bullet \mathbf{v} = c(\mathbf{u} \bullet \mathbf{v})$$

however

$$\mathbf{u} \bullet (c\mathbf{v}) = \overline{c}(\mathbf{u} \bullet \mathbf{v}).$$

So, if you multiply both sides of the equation with the dot product by $\mathbf{u}_i$ on the right, then you can solve for a_i by one division. But if you multiply with dot product on the left, then you first need to factor out $\overline{a_i}$ from the dot product, then solve for $\overline{a_i}$, and finally take the conjugate to get a_i. Either way, you should get the same answer for a_i.)

(f) Find the DFT of the vector

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}.$$

(g) Use these coefficients to write $\mathbf{x}$ in terms of the Fourier basis.