MAT 320 Homework 4
Fall 2024

Due date: Monday, Oct 14

- You can use SciLab, or write a program to help in calculations, for any part of this homework.
- Impulse response always refers to the output y_t of a filter given input $x_t = \delta_t$ where δ is the *Kronecker delta* $(1, 0, 0, \ldots, 0)$. You can also think of this as the first standard basis vector. You may also assume that unless otherwise stated, the values of a signal at negative sample indices are always zero.
- A transfer function of a filter can be written as a function of the delay operator z^{-1} or as a function of the complex variable z. The degree of the transfer function is given by the degree as a polynomial in z^{-1}, so if you make a change of variable $u = z^{-1}$ then you can write as a polynomial in u. So a linear factor of a transfer function is simply a factor which is linear in z^{-1}.

1. Suppose the digital filter in equation 3.1 (Chapter 4, Section 3, page 66) has coefficient $a_1 = 0.98$ and the delay is one sample.
   (a) Sketch a graph of the magnitude response like in figure 3.1, with frequency in fractions of the sampling rate.
   (b) Solve for the max and min values as magnitude and also in dB.
   (c) Plot these points on the graph.

2. Let $y_t = x_t - \frac{3}{2}x_{t-1} - x_{t-2}$ be a filter equation.
   (a) Find the impulse response of this filter.
   (b) Find the transfer function $\mathcal{H}(z)$ of this filter.
   (c) Factor $\mathcal{H}(z)$ into two linear factors $\mathcal{H}_1(z)$ and $\mathcal{H}_2(z)$.
   (d) Find the filter equations of the filters with transfer functions $\mathcal{H}_1(z)$ and $\mathcal{H}_2(z)$.
   (e) Find the impulse response of the cascade of the two filters with transfer functions $\mathcal{H}_1(z)$ and $\mathcal{H}_2(z)$. (See page 70-71.)

3. Let $y_t = x_t - 2x_{t-1} + 2x_{t-2}$ be a filter equation.
   (a) Find the impulse response of this filter.
   (b) Find the transfer function $\mathcal{H}(z)$ of this filter.
   (c) Write this transfer function as a rational function and factor the numerator.
   (d) Write the magnitude response function $|H(\omega)|$
   (e) Compute exact values of the magnitude response for $\omega = 0$, $\omega = \pi/2$, and $\omega = \pi$ using square roots but no decimals.
   (f) Sketch a graph of the magnitude response function.

4. Let $y_t = x_t - 2x_{t-1} + x_{t-2} - 2x_{t-3} + x_{t-4}$ be a filter equation.
   (a) Find the transfer function $\mathcal{H}(z)$ of this filter.
   (b) Find the frequency response function $H(\omega)$ of this filter.
   (c) Find the magnitude response by the method of equation 7.6.
   (d) Find the phase response of this filter as a linear function of ω.