MAT 320 Homework 6
Fall 2020

Due date: Tuesday, Dec 1

You can use SciLab, or write a program to help in calculations, for any part of this homework.

Impulse reponse always refers to the output y_t of a filter given input $x_t = \delta_t = (1, 0, 0, \ldots)$. You may also assume that unless otherwise stated, the values of a signal at negative sample indices are always zero.

1. Suppose a comb filter has variable delay length L. If the sample rate is $f_s = 44100$, how many distinct pitches can be made by adjusting the value of L to produce pitches in the range from A880 to A1760? How many of these pitches are within 5 cents of a pitch on the equal-tempered piano?

2. (a) Reproduce the graph on page 112 of the text book by computing formulas 5.6 and 5.7 yourself. You may use SciLab or any other graphing software. This also includes simply writing a program to compute the input and output values and then plotting these points as a bit map or using another graphing utility.

   (b) Verify by your computations that the tenth harmonic is off only by about 0.027 percent. (Compare the input value of the tenth peak to ten times the input value of the first peak.)

   (c) What does this discrepancy give in terms of Hz, if the sample rate is 44100?

   (d) What is this discrepancy as a frequency ratio?

   (e) What is this discrepancy in terms of cents?

A frequency ratio F_1/F_2 is just that, a ratio of two frequencies. The cent value of such a frequency ratio is defined as:

$$x = \frac{1200}{\ln 2} \ln(F_1/F_2)$$

This formula is chosen so that one octave is divided into 12 chunks of 100 cents each called semitones. On the equal tempered piano there is one semitone between each pair of consecutive keys, black or white. The semitone has the frequency ratio $2^{1/12}$, which gives $x = 100$ cents.

For example, if you compute a frequency value with a comb filter and get frequency 450, how close is this to a frequency of one key on the piano? It is pretty close to A 440. The cent value from 440 to 450 computes as 38.9 cents, so it is not within 5 cents, which is a distance that is quite close to the human ear.