Test ID: TESTID Name: _____

MAT 320 **Midterm Exam** Fall 2020

1. Simplify: $(1-i)^{10}$

   a) $-16e^{-i\pi}$ b) $44\sqrt{2}e^{i\pi}$ c) -64 d) $-32i$ e) $12e^{-i\frac{\pi}{4}}$

   Correct Answer: $-32i$

2. Simplify: $(e^{i\frac{\pi}{3}})^7 + (e^{-i\frac{\pi}{3}})^7$

   a) $\sqrt{3}i$ b) $e^{i2\frac{\pi}{3}}$ c) 1 d) $2e^{i\frac{\pi}{3}}$ e) $3e^{i\frac{\pi}{3}}$

   Correct Answer: 1

3. Simplify: $\cos(\frac{7\pi}{4}) - i\sin(\frac{3\pi}{4})$

   a) $e^{-i\frac{\pi}{4}}$ b) $2e^{i\frac{\pi}{2}}$ c) i d) $e^{i\frac{pi}{4}}$ e) $-i$

   Correct Answer: $e^{-i\frac{\pi}{4}}$

4. Simplify: $\sum_{k=1}^{16}(e^{i\frac{\pi}{8}})^{3k}$

   a) -1 b) $-i$ c) 1 d) i e) 0

   Correct Answer: 0

5. Which function has output values which stay on the unit circle in the complex plane?

   a) $te^{i(2t-1)}$ b) $3e^{i3\pi t}$ c) $(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8})^{2t-3}$ d) $(\cos\frac{\pi}{7} + i\sin\frac{\pi}{7})t^2$ e) $\cos^2 t + \sin^2 t + i(\cos^2 t - \sin^2 t)$

   Correct Answer: $(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8})^{2t-3}$

6. Rotate the complex number $-2+i$ by $\pi/4$ radians counterclockwise, then multiply by $\sqrt{2}$.

   a) $1+2i$ b) $-3-i$ c) i d) $1-3i$ e) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$

   Correct Answer: $-3-i$

7. Write $4i\sin 5t$ as an exponential sum:

   a) $\frac{-3i}{2}\left(e^{i5t} - e^{-i5t}\right)$ b) $2e^{i5t} - 2e^{-i5t}$ c) $4(e^{i5t} - e^{-i5t})$ d) $\frac{2i}{3}\left(e^{i5t} - e^{-i5t}\right)$ e) $ie^{i5t} - ie^{-i5t}$

   Correct Answer: $2e^{i5t} - 2e^{-i5t}$

8. Find the complex dot product of the vectors:

$$(2-i, 1+i) \bullet (1-i, 2+i)$$

   a) $2-6i$ b) $4i$ c) $6+2i$ d) $-2+i$ e) 4

   Correct Answer: $6+2i$

9. Suppose we use sampling rate $f_s = 44100$ Hz. What is the frequency of smallest positive value which is an alias of the frequency 4100 Hz, but is not equal to this frequency?

   a) 2050 b) 22050 c) 2500 d) 4000 e) 40000

   Correct Answer: 40000

10. Let $\{\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2\}$ be the Fourier basis of $\mathbb{C}^3$, where $\mathbf{u}_k$ is the vector obtained by sampling the phasor $e^{i\frac{2\pi}{3}kt}$ at the t-values $0, 1, 2$. What is the sum $\mathbf{u}_0 + \mathbf{u}_1 + \mathbf{u}_2$?

   a) $\begin{pmatrix} 3 \\ e^{-i\frac{2\pi}{3}} \\ e^{-i\frac{4\pi}{3}} \end{pmatrix}$ b) $\begin{pmatrix} 3 \\ e^{i\frac{\pi}{3}} \\ e^{i\frac{2\pi}{3}} \end{pmatrix}$ c) $\begin{pmatrix} 2i \\ i \\ i^2 \end{pmatrix}$ d) $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$ e) $\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$

   Correct Answer: $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

11. Let $\{\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2\}$ be the Fourier basis of $\mathbb{C}^3$, where $\mathbf{u}_k$ is the vector obtained by sampling the phasor $e^{i\frac{2\pi}{3}kt}$ at the t-values $0, 1, 2$. What is $\mathbf{u}_0 \bullet \mathbf{u}_2$?

a) 0 b) i c) 2 d) -2 e) 1

Correct Answer: 0

12. Let $\mathbf{x}$ be a complex vector with coordinates $i, 1, -i, -1$. Find DFT$(\mathbf{x}, 4, 1)$.

a) i b) 0 c) 1 d) -1 e) $-i$

Correct Answer: 0

13. Let $\mathbf{x}$ be a complex vector with coordinates $i, 1, -i, -1$. Find DFT$(\mathbf{x}, 4, 3)$.

a) $-4i$ b) $2i$ c) $4i$ d) -2 e) -4

Correct Answer: $4i$

14. If a filter has frequency response function with peaks (maximum values) at 18 dB, and valleys (minimum values) at -6 dB, then the amplitude range of the signal is varying from low to high by a *factor* of about:

a) 18 b) 16 c) 64 d) 8 e) 4

Correct Answer: 16

15. Suppose that for some filter, a phasor $e^{i\omega_0 t}$ has frequency response given by $H(\omega_0) = e^{i\omega_0/3}$. What is the magnitude response for this frequency given in dB?

a) 6 b) 3 c) 9 d) 0 e) 12

Correct Answer: 0

16. Suppose a filter has magnitude response $|H(\omega)| = |e^{i\omega} - \frac{1}{2}|$. Which frequency ω has the largest frequency response?

a) $\pi/4$ b) $\pi/3$ c) $\pi/2$ d) $2\pi/3$ e) $3\pi/4$

Correct Answer: $3\pi/4$

17. Suppose that for some filter, the input phasor $e^{i\omega_0 t}$ has output phasor $H(\omega_0) \cdot e^{i\omega_0 t}$, where $H(\omega_0) = \frac{e^{i\omega_0/3}}{\sin(\frac{\pi}{4})}$. What is the *magnitude response* $|H(\omega_0)|$ given in dB?

a) -2 b) -1.5 c) 1.5 d) 3 e) -3

Correct Answer: 3

18. Consider the digital filter F_1: $y_t = x_t - \frac{1}{4}y_{t-1}$. What is the transfer function $\mathcal{H}(z)$ for this filter?

a) $1 - \frac{1}{4}z$ b) $1 - \frac{1}{4z}$ c) $\frac{z}{z+\frac{1}{4}}$ d) $\frac{z-\frac{1}{4}}{z}$ e) $1 - 4z$

Correct Answer: $\frac{z}{z+\frac{1}{4}}$

19. Let $f(t) = \frac{4}{\pi}\left[\sin(\omega_0 t) + \frac{1}{3}\sin(3\omega_0 t) + \frac{1}{5}\sin(5\omega_0 t) + \cdots\right]$, with $\omega_0 = 1400\pi$, be the square wave with fundamental (angular) frequency $\omega_0 = \frac{2\pi}{T}$, and period T. The n^{th} harmonic of f is defined to be the term with angular frequency $n\omega_0$, or frequency $700n$ Hz. What is the smallest positive frequency which is an alias of the 150^{th} harmonic, if f is sampled at rate $40,000$ samples per second?

a) 20 kHz b) 10 kHz c) 30 kHz d) 25 kHz e) 15 kHz

Correct Answer: 15 kHz

20. If we change the period of f to be $T = \frac{1}{150}$ of a second, and change ω_0 accordingly, but keep the same sample rate, what is the period of the sampled wave form measured in seconds?

a) $\frac{1}{25}$ b) $\frac{1}{50}$ c) $\frac{1}{20}$ d) $\frac{1}{75}$ e) $\frac{1}{150}$

Correct Answer: $\frac{1}{50}$