

MAT 320

Midterm Exam

Fall 2018

1. Simplify: $(1 + i)^{12}$

- a) $-16e^{-i\pi}$ b) $44\sqrt{2}e^{i\pi}$ c) -64 d) $-18e^{i\pi}$ e) $12e^{-i\frac{\pi}{4}}$

Correct Answer: -64 2. Simplify: $e^{i\frac{\pi}{3}} + (e^{-i\frac{\pi}{3}})^4$

- a) $\sqrt{3}i$ b) $e^{i2\frac{\pi}{3}}$ c) 0 d) $2e^{i\frac{\pi}{3}}$ e) $3e^{i\frac{\pi}{3}}$

Correct Answer: $\sqrt{3}i$ 3. Simplify: $\cos(\frac{7\pi}{4}) + i\sin(\frac{3\pi}{4})$

- a) $e^{i\frac{\pi}{4}}$ b) $2e^{i\frac{\pi}{2}}$ c) i d) $2e^{i\frac{\pi}{4}}$ e) $-i$

Correct Answer: $e^{i\frac{\pi}{4}}$ 4. Simplify: $\sum_{k=1}^{16} (e^{i\frac{\pi}{8}})^{3k}$

- a) -1 b) $-i$ c) 1 d) i e) 0

Correct Answer: 0

5. Which function stays on the unit circle in the complex plane?

- a) $te^{i(2t-1)}$ b) $3e^{i3\pi t}$ c) $(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8})^{2t-3}$ d) $(\cos \frac{\pi}{7} + i \sin \frac{\pi}{7})t^2$
 e) $\cos^2 t + \sin^2 t + i(\cos^2 t - \sin^2 t)$

Correct Answer: $(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8})^{2t-3}$ 6. Rotate the complex number $-2 + i$ by $3\pi/4$ radians counterclockwise, then multiply by $\sqrt{2}$.

- a) $1 + 2i$ b) $-i$ c) i d) $1 - 3i$ e) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$

Correct Answer: $1 - 3i$ 7. Write $2i \sin 3t$ as an exponential sum:

- a) $\frac{-3i}{2}(e^{i3t} - e^{-i3t})$ b) $e^{i3t} - e^{-i3t}$ c) $4(e^{i3t} - e^{-i3t})$ d) $\frac{2i}{3}(e^{i3t} - e^{-i3t})$ e) $ie^{i3t} - ie^{-i3t}$

Correct Answer: $e^{i3t} - e^{-i3t}$

8. Find the complex dot product of the vectors:

$$(1, i) \bullet (1 - i, 2 + i)$$

- a) $2 - i$ b) $4i$ c) $2 + 3i$ d) -2 e) 4

Correct Answer: $2 + 3i$ 9. Suppose we use sampling rate $f_s = 44100$ Hz. What is the frequency of smallest positive value which is an alias of the frequency 4100 Hz, but is not equal to this frequency?

- a) 2050 b) 22050 c) 2500 d) 4000 e) 40000

Correct Answer: 40000

10. Let $\{\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2\}$ be the Fourier basis of \mathbb{C}^3 , where \mathbf{u}_k is the vector obtained by sampling the phasor $e^{i\frac{2\pi}{3}kt}$ at the t -values 0, 1, 2. What is the sum $\mathbf{u}_0 + \mathbf{u}_1 + \mathbf{u}_2$?

a) $\begin{pmatrix} 3 \\ e^{-i\frac{2\pi}{3}} \\ e^{-i\frac{4\pi}{3}} \end{pmatrix}$ b) $\begin{pmatrix} 3 \\ e^{i\frac{\pi}{3}} \\ e^{i\frac{2\pi}{3}} \end{pmatrix}$ c) $\begin{pmatrix} 2i \\ i \\ i^2 \end{pmatrix}$ d) $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$ e) $\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$

Correct Answer: $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

11. Let $\{\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2\}$ be the Fourier basis of \mathbb{C}^3 , where \mathbf{u}_k is the vector obtained by sampling the phasor $e^{i\frac{2\pi}{3}kt}$ at the t -values 0, 1, 2. What is $\mathbf{u}_0 \bullet \mathbf{u}_2$?

a) 0 b) i c) 2 d) -2 e) 1

Correct Answer: 0

12. Let \mathbf{x} be a complex vector with coordinates $i, 1, -i, -1$. Find $\text{DFT}(\mathbf{x}, 4, 1)$.

a) i b) 0 c) 1 d) -1 e) $-i$

Correct Answer: 0

13. Let \mathbf{x} be a complex vector with coordinates $i, 1, -i, -1$. Find $\text{DFT}(\mathbf{x}, 4, 3)$.

a) $-4i$ b) $2i$ c) $4i$ d) -2 e) -4

Correct Answer: $4i$

14. If a filter has frequency response function with peaks (maximum values) at 18 dB, and valleys (minimum values) at -6 dB, then the amplitude range of the signal is varying from low to high by a *factor* of about:

a) 18 b) 16 c) 64 d) 8 e) 4

Correct Answer: 16

15. Suppose that for some filter, a phasor $e^{i\omega_0 t}$ has frequency response given by $H(\omega_0) = e^{i\omega_0/3}$. What is the magnitude response for this frequency given in dB?

a) 6 b) 3 c) 9 d) 0 e) 12

Correct Answer: 0

16. Suppose a filter has magnitude response $|H(\omega)| = |e^{i\omega} - \frac{1}{2}|$. Which frequency ω has the largest frequency response?

a) $\pi/4$ b) $\pi/3$ c) $\pi/2$ d) $2\pi/3$ e) $3\pi/4$

Correct Answer: $3\pi/4$

17. Let $f(t) = \frac{4}{\pi} \left[\sin(\omega_0 t) + \frac{1}{3} \sin(3\omega_0 t) + \frac{1}{5} \sin(5\omega_0 t) + \dots \right]$, with $\omega_0 = 1400\pi$. The n^{th} harmonic of f is defined to be the term with angular frequency $n\omega_0$, or frequency $700n$ Hz. What is the smallest positive frequency which is an alias of the 150^{th} harmonic, if f is sampled at rate 40,000 samples per second?

a) 20 kHz b) 10 kHz c) 30 kHz d) 25 kHz e) 15 kHz

Correct Answer: 15 kHz

18. If we change the period of f to be $T = \frac{1}{150}$ of a second, and change ω_0 accordingly, but keep the same sample rate, what is the period of the sampled wave form measured in seconds?

a) $\frac{1}{25}$ b) $\frac{1}{50}$ c) $\frac{1}{20}$ d) $\frac{1}{75}$ e) $\frac{1}{150}$

Correct Answer: $\frac{1}{50}$