Test ID: TESTID Name: ____________________

MAT 320 **Midterm Exam** Fall 2024

1. Simplify: $(1-i)^{12}$

   a) $-32e^{-i\pi}$ b) $64\sqrt{2}e^{i\pi}$ c) -64 d) $-32(1+i)$ e) $12e^{-i\frac{\pi}{4}}$

   Correct Answer: -64

2. Suppose $z = e^{i\frac{2\pi}{3}}$. Simplify: $z^5 - \overline{z}$, where $\overline{z}$ is the conjugate of z.

   a) 0 b) $e^{i\frac{2\pi}{3}}$ c) 1 d) $2e^{i\frac{2\pi}{3}}$ e) -1

   Correct Answer: 0

3. Simplify: $\cos(\frac{3\pi}{4}) + i\sin(\frac{5\pi}{4})$

   a) $e^{i\frac{5\pi}{4}}$ b) $e^{i\frac{\pi}{4}}$ c) i d) $e^{i\frac{3\pi}{4}}$ e) $-i$

   Correct Answer: $e^{i\frac{5\pi}{4}}$

4. Simplify: $\sum_{k=1}^{31}(e^{i\frac{\pi}{16}})^k$

   a) -1 b) $-i$ c) 1 d) i e) 0

   Correct Answer: -1

5. Which function has output values which are *never* on the unit circle in the complex plane? Consider each function for all real inputs t. (Try sketching the range each expression for all real t and see if it will intersect the unit circle.)

   a) $te^{i(2t-1)}$ b) $e^{i3\pi t}$ c) $(1+i)+(1+2i)t$ d) $i+t$ e) $(1+i)(t-2i)$

   Correct Answer: $(1+i)(t-2i)$

6. Rotate the complex number $1+i$ by $-3\pi/4$ radians counterclockwise, then scale by $\sqrt{2}$.

   a) $1+2i$ b) $-1+3i$ c) $-2i$ d) $-3-i$ e) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$

   Correct Answer: $-2i$

7. Write $\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^2 \sin t$ as an exponential sum:

   a) $\frac{-3i}{2}\left(e^{i3t} - e^{-i3t}\right)$ b) $2e^{it} - 2e^{-it}$ c) $4(e^{it} - e^{-it})$ d) $\frac{2i}{3}\left(e^{i3t} - e^{-i3t}\right)$ e) $e^{it} - e^{-it}$

   Correct Answer: $e^{it} - e^{-it}$

8. Find the complex dot product of the vectors:

$$(i, 1+i) \bullet (1-i, i)$$

   a) $2-i$ b) $1+i$ c) $3+i$ d) $-2+i$ e) 0

   Correct Answer: 0

9. Suppose we use sampling rate $f_s = 16000$ Hz. What is the frequency of smallest positive value which is an alias of the frequency 8100 Hz, but is not equal to this frequency?

   a) 2050 b) 7900 c) 12000 d) 10500 e) 6800

   Correct Answer: 7900

10. Let $\{\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2\}$ be the Fourier basis of $\mathbb{C}^3$, where $\mathbf{u}_k$ is the vector obtained by sampling the phasor $e^{i\frac{2\pi}{3}kt}$ at the t-values $0, 1, 2$. What is the difference $\mathbf{u}_0 + \mathbf{u}_1$?

    a) $\begin{pmatrix} 3 \\ e^{-i\frac{2\pi}{3}} \\ e^{-i\frac{4\pi}{3}} \end{pmatrix}$ b) $\begin{pmatrix} 0 \\ e^{i\frac{\pi}{3}} \\ e^{-i\frac{\pi}{3}} \end{pmatrix}$ c) $\begin{pmatrix} i \\ i \\ i^2 \end{pmatrix}$ d) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ e) $\begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$

    Correct Answer: $\begin{pmatrix} 0 \\ e^{i\frac{\pi}{3}} \\ e^{-i\frac{\pi}{3}} \end{pmatrix}$

11. Let $\{\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2\}$ be the Fourier basis of $\mathbb{C}^3$, where $\mathbf{u}_k$ is the vector obtained by sampling the phasor $e^{i\frac{2\pi}{3}kt}$ at the t-values $0, 1, 2$. What is the complex inner product $\mathbf{u}_2 \bullet \mathbf{u}_2$?

a) 0 b) 1 c) 3 d) -3 e) 2

Correct Answer: 3

12. Let $\mathbf{x}$ be a complex vector with coordinates $1, 0, 1$. Find $X_1 = DFT(\mathbf{x}, 3, 1)$.

a) i b) 1 c) 2 d) $e^{i\frac{\pi}{3}}$ e) $e^{-i\frac{\pi}{3}}$

Correct Answer: $e^{i\frac{\pi}{3}}$

13. Same vector $\mathbf{x}$ and DFT X_1 as in the previous problem. If $\mathbf{x}$ is written in the Fourier basis as $\mathbf{x} = c_0\mathbf{u}_0 + c_1\mathbf{u}_1 + c_2\mathbf{u}_2$ what is c_1?

a) $\frac{1}{3}i$ b) 1 c) $\frac{1}{3}e^{i\frac{\pi}{3}}$ d) $\frac{1}{3}e^{-i\frac{\pi}{3}}$ e) $\frac{2}{3}e^{i\frac{2\pi}{3}}$

Correct Answer: $\frac{1}{3}e^{i\frac{\pi}{3}}$

14. If a filter has frequency response function with peaks (maximum values) at 18 dB, and valleys (minimum values) at -3 dB, then the amplitude range of the signal is varying from low to high by a *factor* of about:

a) 12 b) $8\sqrt{2}$ c) $24\sqrt{2}$ d) $18\sqrt{2}$ e) 16

Correct Answer: $8\sqrt{2}$

15. Suppose that for some filter, the input phasor $e^{i\omega_0 t}$ has output $H(\omega_0)e^{i\omega_0 t}$, where $H(\omega_0) = 2^{\frac{1}{3}}e^{i\omega_0/3}$. What is the magnitude response for this frequency given in dB?

a) 6 b) 3 c) 9 d) 2 e) 0

Correct Answer: 2

16. Suppose a filter has magnitude response $|H(\omega)| = |e^{i\omega} + \frac{i}{2}|$. Which frequency ω has the smallest frequency response?

a) $\pi/4$ b) $\pi/3$ c) $\pi/2$ d) $2\pi/3$ e) $\pi/6$

Correct Answer: $\pi/6$

17. Consider the digital filter: $y_t = 2x_t + \frac{1}{2}x_{t-1} + \frac{1}{2}x_{t-2} + 2x_{t-3}$. What is the transfer function $\mathcal{H}(z)$ for this filter?

a) $1 - 2z$ b) $1 - \frac{1}{2z}$ c) $2 + \frac{1}{2}z^{-1} + \frac{1}{2}z^{-2} + 2z^{-3}$ d) $1 - 2z^{-1} - 2z^{-2} + z^{-3}$ e) $\frac{z^3+2}{z^3}$

Correct Answer: $2 + \frac{1}{2}z^{-1} + \frac{1}{2}z^{-2} + 2z^{-3}$

18. Same filter as in the previous question. Find a simple form for $|H(\omega)|$, the magnitude of the frequency response of this filter.

a) $|\cos(4\omega) + \cos(2\omega)|$ b) $|4\cos(\frac{3}{2}\omega) + \cos(2\omega)|$ c) $|4\cos(\frac{3}{2}\omega) + \cos(\frac{1}{2}\omega)|$ d) $|4\cos(\frac{1}{2}\omega)|$ e) $|4 + 2\cos(\frac{1}{2}\omega)|$

Correct Answer: $|4\cos(\frac{3}{2}\omega) + \cos(\frac{1}{2}\omega)|$

19. Same filter as in the previous question. If the phasor $e^{i\omega t}$ is the input signal, then the output signal will be the same phasor scaled by $|H(\omega)|$ and delayed by how many samples?

a) 1 b) 0.5 c) 1.5 d) 2 e) 2.5

Correct Answer: 1.5

20. The transfer function of a filter, evaluated at a point $e^{i\omega}$ in polar coordinates on the unit circle in the complex plane, gives information in the form of a complex number depending on ω. This complex number is referred to as the filter's:

a) impulse response b) magnitude response c) phase response d) delay e) frequency response

Correct Answer: frequency response