

MAT 321

Quiz 1

Spring 2021

1. Let \mathbf{x} and \mathbf{y} be discrete signals (infinite time extent with values $t \in \mathbb{Z}$) defined by $x_t = 1$ for $t \geq 0$, $x_t = 0$ otherwise, and $y_t = e^{i\frac{\pi}{2}t}$, for $-2 \leq t \leq 2$, $y_t = 0$ otherwise. Find the inner product $\langle \mathbf{x}, \mathbf{y} \rangle$:

a) $-i$ b) i c) 1 d) 0 e) -1

Correct Answer: $-i$

2. Same signals \mathbf{x} and \mathbf{y} as in the previous question. Let \mathbf{w} be the convolution $\mathbf{x} * \mathbf{y}$. Find $\mathbf{w}(0)$ or w_0 .

a) $-i$ b) i c) 1 d) 0 e) -1

Correct Answer: $-i$

3. Let \mathbf{x} be the same discrete signal as in the previous problem, and now let \mathbf{y} be this signal shifted to the right by 2 samples, so that $y_t = x_{t-2}$. What is the Z-transform of the signal \mathbf{y} ?

a) $\frac{z^{-1}}{1-z^{-2}}$ b) $\frac{z^2}{1-z^{-1}}$ c) $\frac{z}{1-z^{-1}}$ d) $\frac{1}{1-z^{-2}}$ e) $\frac{1}{z(z-1)}$

Correct Answer: $\frac{1}{z(z-1)}$

4. Let $b(t)$ be the buzz function defined as: $\sum_{k=-\infty}^{+\infty} \delta(t-k)$ where $\delta(t)$ is the 'delta function' which is zero except for $t = 0$ where it has a concentrated area of value one. What is the value of the integral:

$$\int_{1/2}^{3/2} \sin\left(\frac{\pi}{4}t\right)b(t) dt$$

a) 2 b) $\frac{2}{\pi}$ c) $\sqrt{2}/2$ d) $\sqrt{3}/2$ e) 1

Correct Answer: $\sqrt{2}/2$

5. Solve for a_2 in the following equation:

$$\frac{1}{1+2z^{-1}+3z^{-3}} = a_0 + a_1z^{-1} + a_2z^{-2} + \dots$$

a) 2 b) 3 c) 1 d) 4 e) 0

Correct Answer: 4

6. Let \mathbf{x} be a signal with frequency values $F(\omega)$, where ω is continuous but finite in extent, and let $\mathcal{F}(z) = \sum_{k=-\infty}^{\infty} x_k z^{-k}$ be the Z-transform of \mathbf{x} . The inverse Fourier Transform of F should be computed as:

a) $\frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) e^{i\omega t} d\omega$ b) $\frac{1}{N} \sum_{k=0}^{N-1} X_k e^{i\frac{2\pi}{N}kt}$ c) $\frac{1}{F(\omega)}$ d) $\int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega$ e) $\frac{1}{\mathcal{F}(z)}$

Correct Answer: $\frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) e^{i\omega t} d\omega$

7. Same signal \mathbf{x} as in the previous question. The inverse Z-transform is the process of computing:

a) $\frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) e^{i\omega t} d\omega$ b) x_t from $\mathcal{F}(z)$ c) $\frac{1}{F(\omega)}$ d) $\int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega$ e) $\frac{1}{\mathcal{F}(z)}$

Correct Answer: x_t from $\mathcal{F}(z)$

8. Let \mathbf{x} be a signal which is periodic with continuous time t (finite in extent on an interval of one period). The (forward) Fourier Transform of \mathbf{x} can be thought of as:

a) a value of its Z-transform b) an impulse response c) a delta function d) a coefficient for a phasor
e) an inverse Z-transform

Correct Answer: a coefficient for a phasor

9. Let \mathbf{x} be a signal whose frequency values are discrete of infinite extent. Then \mathbf{x} can be represented as:

- a) a constant b) a step function c) a delta function d) a Fourier series e) an inverse Z -transform

Correct Answer: a Fourier series

10. Which of the following statements are correct in the context of discrete time signals?

- i) The Z -transform of a filter is equal to the transfer function of the filter's impulse response.
ii) The Z -transform of a filter's impulse response is equal to the transfer function of the filter.
iii) The transfer function of a filter is equal to the Z -transform of the filter's magnitude response.

- a) i) only b) i) and ii) only c) ii) only d) iii) only e) ii) and iii) only

Correct Answer: ii) only