

MAT 321

Quiz 1

Spring 2020

1. Let  $\mathbf{x}$  and  $\mathbf{y}$  be discrete signals (infinite time extent with values  $t \in \mathbb{Z}$ ) defined by  $x_t = 1$  for  $t \geq 0$ ,  $x_t = 0$  otherwise, and  $y_t = e^{i\frac{\pi}{2}t}$ , for  $-2 \leq t \leq 2$ ,  $y_t = 0$  otherwise. Find the inner product  $\langle \mathbf{x}, \mathbf{y} \rangle$ :

- a)  $-i$                       b)  $i$                       c)  $1$                       d)  $0$                       e)  $-1$

Correct Answer:  $-i$

2. Same signals  $\mathbf{x}$  and  $\mathbf{y}$  as in the previous question. Let  $\mathbf{w}$  be the convolution  $\mathbf{x} * \mathbf{y}$ . Find  $\mathbf{w}(0)$  or  $w_0$ .

- a)  $-i$                       b)  $i$                       c)  $1$                       d)  $0$                       e)  $-1$

Correct Answer:  $-i$

3. Let  $\mathbf{x}$  be the same discrete signal as in the previous problem, and now let  $\mathbf{y}$  be this signal shifted to the right by 2 samples, so that  $y_t = x_{t-2}$ . What is the Z-transform of the signal  $\mathbf{y}$ ?

- a)  $\frac{z^{-1}}{1-z^{-2}}$                       b)  $\frac{z^2}{1-z^{-1}}$                       c)  $\frac{z}{1-z^{-1}}$                       d)  $\frac{1}{1-z^{-2}}$                       e)  $\frac{1}{z(z-1)}$

Correct Answer:  $\frac{1}{z(z-1)}$

4. Let  $b(t)$  be the buzz function defined as:  $\sum_{k=-\infty}^{+\infty} \delta(t-k)$  where  $\delta(t)$  is the ‘delta function’ which is zero except for  $t = 0$  where it has a concentrated area of value one. What is the value of the integral:

$$\int_{1/2}^{3/2} \sin\left(\frac{\pi}{4}t\right)b(t) dt$$

- a)  $2$                       b)  $\frac{2}{\pi}$                       c)  $\sqrt{2}/2$                       d)  $1$                       e)  $0$

Correct Answer:  $\sqrt{2}/2$

5. Solve for  $a_2$  in the following equation:

$$\frac{1}{1 - 2z^{-1} + 3z^{-3}} = a_0 + a_1z^{-1} + a_2z^{-2} + \dots$$

- a)  $2$                       b)  $3$                       c)  $1$                       d)  $4$                       e)  $0$

Correct Answer:  $4$

6. Let  $\mathbf{x}$  be a signal with frequency values  $F(\omega)$ , where  $\omega$  is continuous but finite in extent, and let  $\mathcal{F}(z) = \sum_{k=-\infty}^{\infty} x_k z^{-k}$  be the  $Z$ -transform of  $\mathbf{x}$ . The inverse Fourier Transform of  $F$  should be computed as:

- a)  $\frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) e^{i\omega t} d\omega$     b)  $\frac{1}{N} \sum_{k=0}^{N-1} X_k e^{i\frac{2\pi}{N} kt}$     c)  $\frac{1}{F(\omega)}$     d)  $\int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega$     e)  $\frac{1}{\mathcal{F}(z)}$

Correct Answer:  $\frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) e^{i\omega t} d\omega$

7. Same signal  $\mathbf{x}$  as in the previous question. The inverse  $Z$ -transform is the process of computing:

- a)  $\frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) e^{i\omega t} d\omega$     b)  $x_t$  from  $\mathcal{F}(z)$     c)  $\frac{1}{F(\omega)}$     d)  $\int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega$     e)  $\frac{1}{\mathcal{F}(z)}$

Correct Answer:  $x_t$  from  $\mathcal{F}(z)$

8. Let  $\mathbf{x}$  be a signal which is periodic with continuous time  $t$  (finite in extent on an interval of one period). The (forward) Fourier Transform of  $\mathbf{x}$  can be thought of as:

- a) a value of its  $Z$ -transform    b) an impulse response    c) a delta function    d) a coefficient for a phasor  
e) an inverse  $Z$ -transform

Correct Answer: a coefficient for a phasor

9. Let  $\mathbf{x}$  be a signal whose frequency values are discrete of infinite extent. Then  $\mathbf{x}$  can be represented as:

- a) a constant    b) a step function    c) a delta function    d) a Fourier series    e) an inverse  $Z$ -transform

Correct Answer: a Fourier series

10. Which of the following statements are correct in the context of discrete time signals?

- i) The  $Z$ -transform of a filter is equal to the transfer function of the filter's impulse response.  
ii) The  $Z$ -transform of a filter's impulse response is equal to the transfer function of the filter.  
iii) The transfer function of a filter is equal to the  $Z$ -transform of the filter's magnitude response.

- a) i) only    b) i) and ii) only    c) ii) only    d) iii) only    e) ii) and iii) only

Correct Answer: ii) only