Test ID: TESTID Name: _____

MAT 321 **Quiz 3** Spring 2023

Unless otherwise indicated, all signals and systems will be assumed to be discrete time (SPW3) with time measured in samples represented by integers. A system or filter is assumed to have an implied domain of all signals for which all outputs are computable, so $\mathbf{x}$ is in the domain of L if its output $\mathbf{y} = L(\mathbf{x})$ is defined for all t by its values y_t which are computed by L. A system or filter with an absolutely summable impulse response will be called ASIR and a system or filter with the property that all bounded input signals produce bounded output signals will be called BIBO. A bounded signal $\mathbf{x}$ is understood to be one for which $|x_t| \leq B_x$ for all t. A signal which is not bounded can be called unbounded, but should not be confused with a signal which cannot be computed.

Recall from calculus that an infinite sum converges to a number M if its sequence of partial sums converges to M. This means that the sequence of partial sums S_n gets closer and closer to M as $n \to \infty$. Divergent sums occur if the values S_n are unbounded, or approaching infinity, or oscillating without approaching a finite value. Typical examples of convergent series are geometric series like $1+r+r^2+r^3+\cdots$ with $|r| < 1$ or alternating series $a_0-a_1+a_2-a_3+-\cdots$ with all $a_i > 0$ and $a_i \to 0$ as $i \to \infty$. Typical examples of divergent series are the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ or an alternating series with general term not approaching zero, such as $\sum_{n=1}^{\infty} \frac{n-1}{n}(-1)^n$

1. Consider the accumulator system:

$$y_t = \sum_{k=-\infty}^{t} x_k.$$

Which of the following input signals $\mathbf{x}$ are NOT in the domain of this system?

i) $x_t = (-1)^t$ ii) $x_t = \frac{(-1)^t}{t}, x_0 = 0.$ iii) $x_t = (-1)^t, t \geq 0,\ x_t = 0, t < 0.$

a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

Correct Answer: i) only

2. Same system as in the previous problem. Which of the following input signals will have output signal which is unbounded?

i) $x_t = \frac{(-1)^t}{t}, x_0 = 0$ ii) $x_t = \frac{(-1)^t}{t}, t > 0,\ x_t = 0, t \leq 0.$ iii) $x_t = \frac{1}{t}, t > 0,\ x_t = 0, t \leq 0.$

a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

Correct Answer: iii) only

3. Same system as in the previous problem. What is the transfer function of this system?

a) $\frac{1}{1-z^{-1}}$ b) $\frac{1}{1-z^{-2}}$ c) $\frac{1}{(1-z^{-1})^2}$ d) $\frac{1}{1-2z^{-1}+z^{-2}}$ e) $1-z^{-1}$

Correct Answer: $\frac{1}{1-z^{-1}}$

4. Same system as in the previous problem. What is the Z-transform of the impulse response of this system?

a) $\frac{1}{1-z^{-1}}$ b) $\frac{1}{1-z^{-2}}$ c) $\frac{1}{(1-z^{-1})^2}$ d) $\frac{1}{1-2z^{-1}+z^{-2}}$ e) $1-z^{-1}$

Correct Answer: $\frac{1}{1-z^{-1}}$

5. Same system as in the previous problem. If the unit step signal is input to this system, what is the Z-transform of the output?

a) $\frac{1}{1-z^{-1}}$ b) $\frac{1}{1-z^{-2}}$ c) $\frac{1}{(1-z^{-1})^2}$ d) $\frac{1}{1-2z^{-1}+z^{-2}}$ e) $1-z^{-1}$

Correct Answer: $\frac{1}{(1-z^{-1})^2}$

6. Let L be the system which computes $y_t = \ln(x_t)$ for any input signal $\mathbf{x}$ with values x_t. If the input $\mathbf{x}$ satisfies $1 < x_t < B$ for some $B > 1$, for all t, then what property is true for the output signal?

   a) Bounded b) Causal c) Undefined d) Absolutely summable e) Unbounded

   Correct Answer: Bounded

7. Which of the following signals are absolutely summable? (Recall that this means that the sum of the absolute values of all the signal values converges.) Assume that each signal is zero for $t \leq 0$.

   i) $x_t = \frac{(-1)^t}{t}$ ii) $x_t = 2^{-t}$ iii) $x_t = 2^t$

   a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

   Correct Answer: ii) only

8. Suppose L is a system which is LTI with impulse response $\mathbf{h}$ and that the domain of L is all signals. Which of the following properties must be true about L?

   i) ASIR ii) BIBO iii) causal

   a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

   Correct Answer: i) and ii) only

9. Suppose L is a system which is LTI with impulse response $\mathbf{h}$ defined as $h_t = 0$ for $t \leq 0$ and $h_t = (-1)^{t+1}\frac{1}{t}$ for $t \geq 1$. Which of the following properties are true about L?

   i) ASIR ii) domain of L is all signals iii) causal

   a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

   Correct Answer: iii) only

10. For this problem we use a translation operator τ_a to indicate that a signal is being delayed by a samples, so that if $\mathbf{w} = \tau_a(\mathbf{x})$ then $w_t = x_{t-a}$. If L is a linear system and an input signal $\mathbf{x}$ has output $\mathbf{y}$, then for L to be an LTI system we would need to know that $L(\tau_a(\mathbf{x}))$ equals:

    a) $\tau_a(\mathbf{x})$ b) $\mathbf{y}$ c) $\tau_a(\mathbf{y})$ d) $\mathbf{x}$ e) $\mathbf{x} + \mathbf{y}$

    Correct Answer: $\tau_a(\mathbf{y})$