

MAT 321

Quiz 4 Answers

Spring 2021

1. If you use integration by parts, and assume that $f(x) \rightarrow 0$ as $x \rightarrow \pm\infty$, then the Fourier Transform of f can be transformed into which of the following? (Use the definition: $\mathcal{F}f(s) = \int_{-\infty}^{\infty} f(t)e^{-2\pi i s t} dt$)

a) $\frac{1}{2\pi i s} (\mathcal{F}f')(s)$ b) $\frac{1}{2\pi i s} (\mathcal{F}f)(s)$ c) $(\mathcal{F}f^2)(s)$ d) $2\pi i s (\mathcal{F}f')(s)$ e) $\frac{1}{2\pi i s} (\mathcal{F}^{-1}f')(s)$

Correct Answer: $\frac{1}{2\pi i s} (\mathcal{F}f')(s)$

2. What is the Fourier Transform of the distribution $1 + \delta$?

a) 0 b) $1 - \delta$ c) $1 + \delta$ d) 1 e) e^{-t}

Correct Answer: $1 + \delta$

3. Let f be the function which is 1 for $t < 1$ and 3 for $t > 1$. Find the derivative of f as a distribution.

a) $2\delta_1$ b) 3δ c) 2δ d) $3\delta_1$ e) $\delta_1 + \delta_{-1}$

Correct Answer: $2\delta_1$

4. Suppose T is a tempered distribution which assigns to a Schwartz function its value at $t = 1$ minus its value at $t = -1$. Choose an equivalent form for T :

a) $\sin(t-1) - \sin(t+1)$ b) $\cos(t-1) - \cos(t+1)$ c) $\frac{1}{1-t}$ d) $\delta_1 - \delta_{-1}$ e) $e^{i\frac{t}{2}} - e^{-i\frac{t}{2}}$

Correct Answer: $\delta_1 - \delta_{-1}$

5. Let $h_p(t)$ be the function with value $\frac{1}{p}$ for $-\frac{p}{2} \leq t \leq \frac{p}{2}$, and zero elsewhere. Let T_{h_p} be the distribution defined by integrating against a Schwartz function in the usual way:

$$\langle T_{h_p}, \phi \rangle = \int_{-\infty}^{\infty} h_p(t)\phi(t) dt.$$

Find the limit of the distributions: $\lim_{p \rightarrow 0} T_{h_p}$.

a) $\sin t$ b) $\cos t$ c) $\frac{1}{1-t}$ d) δ e) $e^{i\frac{t}{2}} - e^{-i\frac{t}{2}}$

Correct Answer: δ

6. Suppose T assigns to a Schwartz function its value at $t = 0$ squared, ie. T is defined by the pairing: $\langle T, \phi \rangle = \phi(0)^2$. Then T fails to be a tempered distribution for which of the following reasons?

i) T is not linear ii) T is not continuous iii) T is not defined by an integral

a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

Correct Answer: i) only

7. Let $f_p(t)$ be the function with value $1 - 1/p$ for $-p \leq t \leq p$, and zero elsewhere. Let T_{f_p} be defined by integrating against a Schwartz function in the usual way:

$$\langle T_{f_p}, \phi \rangle = \int_{-\infty}^{\infty} f_p(t)\phi(t) dt.$$

Let T be the limit of the distributions: $\lim_{p \rightarrow \infty} T_{f_p}$. What is the Fourier Transform of T , ie. $\mathcal{F}T$?

a) 1 b) $2\cos(2\pi t)$ c) $\frac{1}{1-t}$ d) δ e) $e^{i\frac{t}{2}} - e^{-i\frac{t}{2}}$

Correct Answer: δ

8. Denote the following operators on distributions or functions: D means take the derivative, \mathcal{F} means take the Fourier Transform, τ_b means shift right by b , σ_a means scale the argument by a , M means apply the minus operator. (Most of these are just as we used them in class, except for D and M . In those cases we mean, for instance, $D(f) = f'$, or $D(T) = T'$, and $M(f) = f^-$ or $M(T) = T^-$.) Which of these operators X have the very simple form for distribution T :

$$\langle MT, \phi \rangle = \langle T, M\phi \rangle$$

- a) all of them b) \mathcal{F} and M only c) τ_b and σ_a only d) D only e) D and \mathcal{F} only

Correct Answer: \mathcal{F} and M only