MAT 321   **Quiz 4**   Spring 2024

The following abbreviations will be used on this quiz: r.v.e. (real-valued and even), r.v.o. (real-valued and odd), i.v.o. (imaginary-valued and odd), i.v.e. (imaginary-valued and even). Signals in time domain will typically be given names like f or g and their Fourier Transforms (in any SPW) names like $\mathcal{F}f$ or $\mathcal{F}g$.

1. Let $f(t) = \cos t$ be a time domain signal in SPW2. Its Fourier Transform $\mathcal{F}f$ can be written as a discrete valued frequency signal with parameter $k \in \mathbb{Z}$. Based on the properties of f and time-frequency correspondences (duality), we can conclude that $\mathcal{F}f$ is

   a) r.v.e.   b) r.v.o.   c) i.v.o.   d) i.v.e.   e) none of these

   Correct Answer: r.v.e.

2. Same f and $\mathcal{F}f$ as in the previous problem. Choose the correct portion of the signal $\mathcal{F}f$ for k values from -2 to 2:

   a) $(0, -\frac{1}{2}, 0, \frac{1}{2}, 0)$ b) $(0, \frac{1}{2}, 0, -\frac{1}{2}, 0)$ c) $(0, \frac{1}{2}, 0, \frac{1}{2}, 0)$ d) $(0, \frac{1}{2}i, 0, \frac{1}{2}i, 0)$ e) $(0, -\frac{1}{2}i, 0, \frac{1}{2}i, 0)$

   Correct Answer: $(0, \frac{1}{2}, 0, \frac{1}{2}, 0)$

3. Suppose g is an analog signal, with Fourier Transform $\mathcal{F}g$, which is sampled at f_s Hz, or angular frequency ω_s, to produce a digital signal $\mathbf{x}$. Nyquist's Theorem gives conditions under which $\mathbf{x}$ can be used, together with shifted versions of the function $\text{sinc}(x)$, to perfectly reconstruct f. Which inequality best describes this condition?

   a) $-\frac{\omega_s}{2} \leq \mathcal{F}g \leq \frac{\omega_s}{2}$ b) $-\omega_s \leq \mathcal{F}g \leq \omega_s$ c) $-\omega_s \leq \mathbf{x} \leq \omega_s$ d) $-\frac{f_s}{2} \leq \mathbf{x} \leq \frac{f_s}{2}$ e) $-f_s \leq \text{sinc}(x) \leq f_s$

   Correct Answer: $-\frac{\omega_s}{2} \leq \mathcal{F}g \leq \frac{\omega_s}{2}$

4. The function $\text{sinc}(x)$ uses the removable discontinuity in the formula $\dfrac{\sin(\pi x)}{\pi x}$, in order to define its value at $x = 0$ to be:

   a) π b) 0   c) 2π d) 1   e) -1

   Correct Answer: 1

5. Suppose f is a time domain signal in SPW1 with Fourier Transform $\mathcal{F}f$. If f is even then we can conclude that

   a) $\mathcal{F}f = \mathcal{F}^{-1}f$ b) $(\mathcal{F}f)^- = -\mathcal{F}f$ c) $f = \mathcal{F}^{-1}f$ d) $\mathcal{F}f$ is even   e) $f^- = \mathcal{F}^{-1}f$

   Correct Answer: $\mathcal{F}f$ is even

6. Suppose f is a time domain signal in SPW1 with Fourier Transform $\mathcal{F}f$. If f is real-valued then we can conclude that

a) $\overline{\mathcal{F}f} = \mathcal{F}f^-$ 　　b) $\overline{\mathcal{F}f} = -\mathcal{F}f$ 　　c) $\overline{\mathcal{F}f} = f$ 　　d) $\mathcal{F}f$ is real-valued 　　e) $\mathcal{F}f$ is imaginary-valued

Correct Answer: $\overline{\mathcal{F}f} = \mathcal{F}f^-$

7. Suppose f is a time domain signal in SPW1 with Fourier Transform $\mathcal{F}f$. Then $\mathcal{F}\mathcal{F}f$ is equivalent to:

a) Nf^- 　　　　b) N^2f^- 　　　　c) $\mathcal{F}^{-1}f$ 　　　　d) $\mathcal{F}f^-$ 　　　　e) $N\mathcal{F}f^-$

Correct Answer: Nf^-

8. Suppose $f(t) = i\sin(t)$ is a time domain signal in SPW2 with Fourier Transform $\mathcal{F}f$. Then what can be said about $\mathcal{F}f$?

a) r.v.e. 　　　　b) r.v.o. 　　　　c) i.v.o. 　　　　d) i.v.e. 　　　　e) none of these

Correct Answer: r.v.o.

9. Same f and $\mathcal{F}f$ as in the previous problem. Choose the correct portion of the signal $\mathcal{F}f$ for k values from -2 to 2:

a) $(0, -\frac{1}{2}, 0, \frac{1}{2}, 0)$ 　　b) $(0, \frac{1}{2}, 0, -\frac{1}{2}, 0)$ 　　c) $(0, \frac{1}{2}, 0, \frac{1}{2}, 0)$ 　　d) $(0, \frac{1}{2}i, 0, \frac{1}{2}i, 0)$ 　　e) $(0, -\frac{1}{2}i, 0, \frac{1}{2}i, 0)$

Correct Answer: $(0, -\frac{1}{2}, 0, \frac{1}{2}, 0)$

10. Let $f(t) = \dfrac{\sin(\pi t)}{\pi t}$, for $t \neq 0$, and $f(0) = 1$, be a time domain signal in SPW3, where t is an integer variable representing sample numbers. What is the best description of this signal in words?

a) impulse response 　b) frequency response 　c) transfer function 　d) delta pulse 　e) discrete impulse

Correct Answer: discrete impulse