MAT 321 Practice Quiz 5 Answer Sheet

Spring 2022

Quiz ID: TESTID Name: ____________________

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2. ☐
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MAT 321 **Practice Quiz 5** Spring 2022

1. If you use integration by parts, and assume that $f(x) \longrightarrow 0$ as $x \longrightarrow \pm\infty$, then the Fourier Transform of f can be transformed into which of the following? (Use the definition: $\mathcal{F}f(s) = \int_{-\infty}^{\infty} f(t)e^{-2\pi ist}\, dt$)

   a) $\frac{1}{2\pi is}\left(\mathcal{F}f'\right)(s)$ b) $\frac{1}{2\pi is}\left(\mathcal{F}f\right)(s)$ c) $\left(\mathcal{F}f^2\right)(s)$ d) $2\pi is\left(\mathcal{F}f'\right)(s)$ e) $\frac{1}{2\pi is}\left(\mathcal{F}^{-1}f'\right)(s)$

2. What is the Fourier Transform of the distribution $1+\delta$?

   a) 0 b) $1-\delta$ c) $1+\delta$ d) 1 e) e^{-t}

3. Let f be the function which is 1 for $t<1$ and 3 for $t>1$. Find the derivative of f as a distribution.

   a) $2\delta_1$ b) 3δ c) 2δ d) $3\delta_1$ e) $\delta_1+\delta_{-1}$

4. Suppose T is a tempered distribution which assigns to a Schwartz function its value at $t=1$ minus its value at $t=-1$. Choose an equivalent form for T:

   a) $\sin(t-1)-\sin(t+1)$ b) $\cos(t-1)-\cos(t+1)$ c) $\frac{1}{1-t}$ d) $\delta_1-\delta_{-1}$ e) $e^{i\frac{t}{2}}-e^{-i\frac{t}{2}}$

5. Let $h_p(t)$ be the function with value $\frac{1}{p}$ for $-\frac{p}{2}\le t\le\frac{p}{2}$, and zero elsewhere. Let T_{h_p} be the distribution defined by integrating against a Schwartz function in the usual way:

   $$\langle T_{h_p},\phi\rangle = \int_{-\infty}^{\infty} h_p(t)\phi(t)\, dt.$$

   Find the limit of the distributions: $\lim_{p\to 0} T_{h_p}$.

   a) $\sin t$ b) $\cos t$ c) $\frac{1}{1-t}$ d) δ e) $e^{i\frac{t}{2}}-e^{-i\frac{t}{2}}$

6. Suppose T assigns to a Schwartz function its value at $t=0$ squared, ie. T is defined by the pairing: $\langle T,\phi\rangle = \phi(0)^2$. Then T fails to be a tempered distribution for which of the following reasons?

   i) T is not linear ii) T is not continuous iii) T is not defined by an integral

   a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

7. Let $f_p(t)$ be the function with value $1-1/p$ for $-p\le t\le p$, and zero elsewhere. Let T_{f_p} be defined by integrating against a Schwartz function in the usual way:

   $$\langle T_{f_p},\phi\rangle = \int_{-\infty}^{\infty} f_p(t)\phi(t)\, dt.$$

   Let T be the limit of the distributions: $\lim_{p\to\infty} T_{f_p}$. What is the Fourier Transform of T, ie. $\mathcal{F}T$?

   a) 1 b) $2\cos(2\pi t)$ c) $\frac{1}{1-t}$ d) δ e) $e^{i\frac{t}{2}}-e^{-i\frac{t}{2}}$

8. Denote the following operators on distributions or functions: D means take the derivative, $\mathcal{F}$ means take the Fourier Transform, τ_b means shift right by b, σ_a means scale the argument by a, M means apply the minus operator. (Most of these are just as we used them in class, except for D and M. In those cases we mean, for instance, $D(f)=f'$, or $D(T)=T'$, and $M(f)=f^-$ or $M(T)=T^-$.) Which of these operators X have the very simple form for distribution T:

   $$< MT,\phi > = < T, M\phi >$$

   a) all of them b) $\mathcal{F}$ and M only c) τ_b and σ_a only d) D only e) D and $\mathcal{F}$ only