

MAT 321

Quiz 5

Spring 2024

Note: $L^1(\mathbb{R})$ is the set of functions f for which $\int_{-\infty}^{\infty} |f(x)| dx$ exists.

1. Suppose T assigns to a Schwartz function its value at $t = 0$ plus its value at $t = 1$. Which of the following are true statements about T ?

i) T is linear ii) T is a distribution iii) T is a sum of deltas

a) i) and ii) only b) i) and iii) only c) iii) only d) all of them e) i) only

Correct Answer: all of them

2. Let $h_k(t)$ be the function with value $1/k$ for $-\frac{k}{2} \leq t \leq \frac{k}{2}$, and zero elsewhere. Let T_{h_k} be defined by integrating against a Schwartz function in the usual way:

$$\langle T_{h_k}, \phi \rangle = \int_{-\infty}^{\infty} h_k(t) \phi(t) dt.$$

Let T be the limit of the distributions: $\lim_{k \rightarrow 0} T_{h_k}$. What is the Fourier Transform of T , ie. $\mathcal{F}T$?

a) 1 b) $2 \cos(2\pi s)$ c) $\frac{1}{1-s}$ d) δ e) $e^{-2\pi i s}$

Correct Answer: 1

3. Let $f(t) = \frac{1}{1+t^2}$ and let $g(t) = t$. Which of the following are in $L^1(\mathbb{R})$?

i) $f(t)$ ii) $g(t)$ iii) $f(t)g(t)$

a) all of them b) i) only c) ii) and iii) only d) i) and ii) only e) none of them

Correct Answer: i) only

4. Same functions f , g , and choices i), ii), iii). Which of the choices are Schwartz functions?

a) all of them b) i) only c) ii) and iii) only d) i) and ii) only e) none of them

Correct Answer: none of them

5. Find the Fourier Transform of the distribution $1 + \delta$:

a) $1 + \delta$ b) 1 c) 2 d) δ e) δ_1

Correct Answer: $1 + \delta$

6. Find the integral $\int_0^1 1 + \delta dx$:

a) $1 + \delta$ b) 1 c) 2 d) δ e) δ_1

Correct Answer: 2

7. Let f be the function which is equal to -1 for $0 < t < 1$ and 1 for $t > 1$ and 0 elsewhere. Find the derivative of f as a distribution:

a) $1 + \delta_1$ b) 2 c) $2\delta_1 - \delta$ d) $\delta - 1$ e) δ_1

Correct Answer: $2\delta_1 - \delta$

8. Let T be defined by $\langle T, \phi \rangle = \phi(0)^2$ for any Schwartz function ϕ . Suppose f is the rectangle function $f(t) = 1$ for $-\frac{1}{2} \leq t \leq \frac{1}{2}$, and is zero elsewhere. Let $g(t) = f(t-1)$. Find $\langle T, f+g \rangle$:

a) 0 b) 2 c) 1 d) $\frac{1}{2}$ e) $-\frac{1}{2}$

Correct Answer: 1

9. Same T , f , and g as in the previous question. Determine if the statements about T are True or False:

- i) T is linear ii) T is a distribution iii) $\langle T, f + g \rangle = \langle T, f \rangle + \langle T, g \rangle$
a) TTT b) FFT c) FFF d) FTT e) TFT

Correct Answer: FFT

10. Find the Fourier Transform (as a distribution) of $\sin(2\pi x) + i \cos(2\pi x)$.

- a) δ_1 b) $i\delta_{-1}$ c) $\delta - \delta_1$ d) $i\delta - i\delta_1$ e) $\delta_{-1} + i\delta_1$

Correct Answer: $i\delta_{-1}$