Test ID: TESTID Name: ____________________

MAT 321 **Quiz 6** Spring 2026

Note: $L^1(\mathbb{R})$ is the set of functions f for which $\int_{-\infty}^{\infty} |f(x)|\, dx$ exists.

1. Suppose T assigns to a Schwartz function its value at $t = 0$ plus its value at $t = 1$. Which of the following are true statements about T?

   i) T is linear ii) T is a distribution iii) T is a sum of deltas

   a) i) and ii) only b) i) and iii) only c) iii) only d) all of them e) i) only

   Correct Answer: all of them

2. Let $h_k(t)$ be the function with value $1/k$ for $-\frac{k}{2} \le t \le \frac{k}{2}$, and zero elsewhere. Let T_{h_k} be defined by integrating against a Schwartz function in the usual way:

$$\langle T_{h_k}, \phi \rangle = \int_{-\infty}^{\infty} h_k(t)\phi(t)\, dt.$$

   Let T be the limit of the distributions: $\lim_{k \to 0} T_{h_k}$. What is the Fourier Transform of T, ie. $\mathcal{F}T$?

   a) 1 b) $2\cos(2\pi s)$ c) $\frac{1}{1-s}$ d) δ e) $e^{-2\pi i s}$

   Correct Answer: 1

3. Let $f(t) = \dfrac{1}{1+t^2}$ and let $g(t) = t$. Which of the following are in $L^1(\mathbb{R})$? (Recall: an antiderivative of $f(t)$ is $\tan^{-1}(t)$, and an antiderivative of $f(t)g(t)$ can be found by u-substitution.)

   i) $f(t)$ ii) $g(t)$ iii) $f(t)g(t)$

   a) all of them b) i) only c) ii) and iii) only d) i) and ii) only e) none of them

   Correct Answer: i) only

4. Same functions f, g, and choices i), ii), iii). Which of the choices are Schwartz functions? (Recall: a Schwartz function needs to be rapidly decreasing, at least as fast as $1/e^x$.)

   a) all of them b) i) only c) ii) and iii) only d) i) and ii) only e) none of them

   Correct Answer: none of them

5. Find the Fourier Transform of the distribution $1 + \delta$:

   a) $1 + \delta$ b) 1 c) 2 d) δ e) δ_1

   Correct Answer: $1 + \delta$

6. Find the integral $\int_0^1 1 + \delta \; dx$:

a) $1 + \delta$ b) 1 c) 2 d) δ e) δ_1

Correct Answer: 2

7. Let f be the function which is equal to -1 for $0 < t < 1$ and 1 for $t > 1$ and 0 elsewhere. Find the derivative of f as a distribution:

a) $1 + \delta_1$ b) 2 c) $2\delta_1 - \delta$ d) $\delta - 1$ e) δ_1

Correct Answer: $2\delta_1 - \delta$

8. Let T be defined by $< T, \phi >= \phi(0)^2$ for any Schwartz function ϕ. Suppose f is the rectangle function $f(t) = 1$ for $-\frac{1}{2} \leq t \leq \frac{1}{2}$, and is zero elsewhere. Let $g(t) = f(t-1)$. (Then f and g are Schwartz functions.) Find $< T, f + g >$:

a) 0 b) 2 c) 1 d) $\frac{1}{2}$ e) -$\frac{1}{2}$

Correct Answer: 1

9. Same T, f, and g as in the previous question. Determine if the statements about T are True or False: (Note: iii) is a statement about specific functions f and g defined above. i) and ii) are general statements which need to apply to all possible inputs.)

i) T is linear ii) T is a distribution iii) $< T, f + g >=< T, f > + < T, g >$

a) TTT b) FFT c) FFF d) FTT e) TFT

Correct Answer: FFT

10. Choose an equivalent formula for the integral:

$$\int_{-\infty}^{\infty} e^{2\pi i a x} \mathcal{F}\phi(x) \; dx.$$

a) $\left(\mathcal{F}^{-1}\phi\left(\mathcal{F}\phi\right)\right)(a)$ b) $\left(\mathcal{F}\phi(a)\left(\mathcal{F}^{-1}\phi\right)\right)(a)$ c) $\left(\mathcal{F}^{-1}\left(\mathcal{F}\phi\right)\right)(a)$ d) $\mathcal{F}^{-1}\left(\phi(a)\right)$ e) $\mathcal{F}\left(\phi(a)\right)$

Correct Answer: $\left(\mathcal{F}^{-1}\left(\mathcal{F}\phi\right)\right)(a)$