MAT 321 Homework 4
Spring 2019

Due date: Tuesday, February 19

Let $\mathcal{F}$ denote the Fourier transform and $\mathcal{F}^{-1}$ its inverse, so that (as in the notes of Osgood):

$$\mathcal{F}f(s) = \int_{-\infty}^{\infty} f(t)e^{-i2\pi st}\,dt \qquad \text{and} \qquad \mathcal{F}^{-1}f(t) = \int_{-\infty}^{\infty} f(s)e^{i2\pi st}\,ds.$$

We established the following identities and implications in class for the continuous, infinite extent domains. Note: we define f^- to be the function given by $f^-(t) = f(-t)$. Also, the exponent minus is assumed to apply directly to its left. If it applies to more than one symbol, we use parentheses. For example $(\mathcal{F}f)^-$ can be thought of as the function given by $g^-(s) = g(-s)$, where $g = \mathcal{F}f$.

1. $(\mathcal{F}f)^- = \mathcal{F}^{-1}f$
2. $\left(\mathcal{F}^{-1}f\right)^- = \mathcal{F}f$
3. $\mathcal{F}f^- = \mathcal{F}^{-1}f$
4. $\mathcal{F}f^- = (\mathcal{F}f)^-$
5. $\mathcal{F}^{-1}f^- = \left(\mathcal{F}^{-1}f\right)^-$
6. $\mathcal{F}^{-1}f^- = \mathcal{F}f$
7. $\mathcal{F}\mathcal{F}f = f^-$
8. f even $\iff$ $\mathcal{F}f$ even (or equivalently: f even $\iff$ $\mathcal{F}^{-1}f$ even)
9. f odd $\iff$ $\mathcal{F}f$ odd (or equivalently: f odd $\iff$ $\mathcal{F}^{-1}f$ odd)
10. f real-valued $\implies$ $(\mathcal{F}f)^- = \overline{\mathcal{F}f} = \mathcal{F}f^-$ (same for $\mathcal{F}^{-1}$)
11. f imaginary-valued $\implies$ $(\mathcal{F}f)^- = -\overline{\mathcal{F}f} = \mathcal{F}f^-$ (same for $\mathcal{F}^{-1}$)
12. f real-valued and even $\iff$ $\mathcal{F}f$ real-valued and even (same for $\mathcal{F}^{-1}$)
13. f imaginary-valued and even $\iff$ $\mathcal{F}f$ imaginary-valued and even (same for $\mathcal{F}^{-1}$)
14. f real-valued and odd $\iff$ $\mathcal{F}f$ imaginary-valued and odd (same for $\mathcal{F}^{-1}$)

Investigate each of these implications and identities for the case of finite domains. In this case the time and frequency domains are still symmetric, both being finite. The variables are t and k, instead of t and s.

For each statement, first determine the analogous statement for the finite domains, then decide if this new statement is true, or if it needs to be modified to make it true. For instance, you might need a factor N or $1/N$ to make it true. Finally, give the derivation of each true statement using the DFT and inverse DFT, including a factor of N or $1/N$ wherever necessary.