MAT 321 Homework 5
Spring 2019

Due date: Tuesday, March 26

Let $\mathcal{F}$ denote the Fourier transform and $\mathcal{F}^{-1}$ its inverse, so that (as in the notes of Osgood):

$$\mathcal{F}f(s) = \int_{-\infty}^{\infty} f(t)e^{-i2\pi st}\,dt \qquad \text{and} \qquad \mathcal{F}^{-1}f(t) = \int_{-\infty}^{\infty} f(s)e^{i2\pi st}\,ds.$$

1. Show that $(\mathcal{F}f')(s) = 2\pi i s\mathcal{F}f(s)$ (Use integration by parts.)

2. Let $f(t) = e^{-at}$, for $t \geq 0$, and $f(t) = 0$, for $t < 0$, where $a > 0$. Find the Fourier transform $\mathcal{F}f(s)$.

3. Let $g(t) = f(t) + f^{-}(t)$, for $t \neq 0$, and $g(0) = 1$, where f is the same function as in the previous problem. Find the Fourier transform $\mathcal{F}g(s)$.

4. Show that convolution of functions is associative: $f * (g * h) = (f * g) * h$.

5. Find the second and third derivatives of the delta distribution.