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MAT 321 **Midterm Exam** Spring 2026

1. Let $\mathbf{x}$ and $\mathbf{y}$ be discrete time signals (infinite time extent with values $t \in \mathbb{Z}$) defined by $x_t = t$ for all t, and $y_t = e^{i\frac{\pi}{2}t}$, for $0 \le t \le 2$, $y_t = 0$ otherwise. Find the inner product $< \mathbf{x}, \mathbf{y} >$:

   a) $2 - i$ b) $2 + 2i$ c) $-2 + 2i$ d) $-2 + i$ e) $-2 - i$

   Correct Answer: $-2 - i$

2. Same signals $\mathbf{x}$ and $\mathbf{y}$ as in the previous question. Let $\mathbf{w}$ be the convolution $\mathbf{x} * \mathbf{y}$. Find $\mathbf{w}(1)$ or w_1.

   a) $1 - i$ b) $1 + 2i$ c) $-1 + 2i$ d) $-1 + i$ e) $1 - 2i$

   Correct Answer: 2

3. Let $\mathbf{y}$ be the signal with values $y_t = 1$ for $t > 0$, t an odd integer, and 0 elsewhere. What is the Z-transform of the signal $\mathbf{y}$?

   a) $\frac{z^{-3}}{(1-z^{-1})^2}$ b) $\frac{z^2}{1-z^{-1}}$ c) $\frac{1}{1-z^{-3}}$ d) $\frac{z^{-1}}{1-z^{-2}}$ e) $\frac{1}{(z-1)^3}$

   Correct Answer: $\frac{z^{-1}}{1-z^{-2}}$

4. Suppose a signal with frequency values $X(\omega)$ is given as:

$$x_t = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{i\omega t} \, d\omega.$$

   Then the time domain can be expressed as:

   i) discrete ii) finite extent iii) continuous iv) infinite extent

   a) i) and iv) only b) i) and ii) only c) ii) and iii) only d) iii) and iv) only e) i) only

   Correct Answer: i) and iv) only

5. Solve for a_1 in the following equation:

$$\frac{1}{1 - z^{-1} + 2z^{-2}} = a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots .$$

   a) 2 b) -1 c) 1 d) -2 e) 0

   Correct Answer: 1

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6. Suppose the signal $\mathbf{x}$ is a discrete time phasor sampled for all time t with values $e^{i\frac{\pi}{2}t}$. The inner product of $\mathbf{x}$ with itself can be best described as:

   a) a delta spike b) finite c) an impulse response d) 1 e) zero

   Correct Answer: a delta spike

7. Let $\mathbf{x}$ be a signal whose time values are discrete of infinite extent. Then the possible frequency values present in the spectrum of this signal can be represented as:

   a) $k\omega_0$, $k \in \mathbb{Z}$ b) an interval $[-\pi, \pi)$ c) all real numbers $\mathbb{R}$ d) $k2\pi/N$, $k = 0, \ldots, N-1$ e) a Fourier series

   Correct Answer: an interval $[-\pi, \pi)$

8. Simplify: $|1 - e^{-i\omega}|^{-2}$

   a) $\frac{1}{1-\sin\omega}$ b) $\frac{1}{1+\sin^2\omega}$ c) $\frac{1/2}{1+\cos^2\omega}$ d) $\frac{1/2}{1-\cos\omega}$ e) $\frac{2}{1+\cos^2\omega}$

   Correct Answer: $\frac{1/2}{1-\cos\omega}$

9. Consider the following system (or filter), with domain of input signals understood to be those signals $\mathbf{x}$ for which the output $\mathbf{y}$ is defined for all t:

$$y_t = \sum_{k=1}^{t} \frac{x_k}{k},\ t \geq 1, \qquad y_t = x_t,\ t < 1.$$

   (Assume the definition holds for all t, unless indicated otherwise.) Which of the following input signals satisfy at lease one of the properties: i) NOT in the domain of this system, or ii) not Bounded?

   i) $x_t = (-1)^t$ ii) $x_t = 1$ iii) $x_t = t, t \geq 0,\ x_t = 0, t < 0.$

   a) i) and ii) only b) none of them c) iii) only d) ii) only e) i) only

   Correct Answer: iii) only

10. Same system and same input signals as in the previous problem. Which of the following input signals satisfy both of the properties: i) the input signal is bounded, and ii) the associated output signal in unbounded?

    a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

    Correct Answer: ii) only

11. Same system as in the previous problem. Let $\boldsymbol{\delta}$ be the unit impulse signal, and let $\boldsymbol{\delta}_1$ be the unit impulse shifted by 1 sample to the right. Also let $\mathbf{u}$ be the unit step signal, and let $\mathbf{u}_1$ be the unit impulse shifted by 1 sample to the right. Also let $\mathbf{v}$ be the sum $\boldsymbol{\delta} + \boldsymbol{\delta}_1$. What is the impulse response of this system?

a) $\mathbf{u}$ b) $\boldsymbol{\delta}$ c) $\boldsymbol{\delta}_1$ d) $\mathbf{u}_1$ e) $\mathbf{v}$

Correct Answer: $\boldsymbol{\delta}$

12. Same system and signals as in the previous problem. What is the output of this system with input $\boldsymbol{\delta}_1$?

a) $\mathbf{u}$ b) $\boldsymbol{\delta}$ c) $\boldsymbol{\delta}_1$ d) $\mathbf{u}_1$ e) $\mathbf{v}$

Correct Answer: $\mathbf{u}_1$

13. Same system as in the previous problem. Which of the following properties are true for this system? (Recall that causal means values of the output are computed only using the current input and prior inputs.)

i) linear ii) time invariant iii) causal

a) i) and ii) only b) ii) and iii) only c) i) and iii) only d) i) only e) iii) only

Correct Answer: i) and iii) only

14. Let $\mathbf{r}$ be the convolution signal $\mathbf{x} * \mathbf{x}$. If $\mathbf{x}$ has frequency content $X(\omega)$, then $\mathbf{r}$ must have frequency content:

a) $X(\omega)^2$ b) $X(\omega^2)$ c) $X(2\omega)$ d) $2X(\omega)$ e) none of these

Correct Answer: $X(\omega)^2$

15. Suppose a filter with transfer function $\mathcal{H}(z)$ has impulse response $\mathbf{h}$ given by $h_t = 0$ for $t \leq 0$ and $h_t = \dfrac{(-1)^t}{t}$ for $t > 0$. Which of the following properties are true for $\mathbf{h}$?

i) absolutely convergent ii) conditionally convergent iii) convergent

a) i) and ii) only b) ii) and iii) only c) i) and iii) only d) i) only e) iii) only

Correct Answer: ii) and iii) only

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16. Suppose a filter with transfer function $\mathcal{H}(z)$ has impulse response $\mathbf{h}$ given by $h_t = 0$ for $t < 0$ and $h_t = (-1)^t e^{-t}$ for $t \geq 0$. Which of the following properties are true for $\mathbf{h}$?

    i) absolutely convergent ii) conditionally convergent iii) convergent

    a) i) and ii) only b) ii) and iii) only c) i) and iii) only d) i) only e) iii) only

    Correct Answer: i) and iii) only

17. A raised cosine window $f(t) = a - b\cos(2\pi t)$ can be designed on the interval $[0, 1]$ to have maximum value 1.0 and minimum value 0.2 for which value of a?

    a) 0.5 b) 0.6 c) 0.7 d) 0.4 e) 0.3

    Correct Answer: 0.6

18. Same raised cosine window $f(t) = a - b\cos(2\pi t)$ as in the previous problem. What is the area under this window graph and above the t axis over the interval $[0, 1]$?

    a) 0.5 b) 0.6 c) 0.7 d) 0.4 e) 0.3

    Correct Answer: 0.6

19. Let $\mathbf{x}$ be a signal which is periodic with continuous time t on an interval $[a, b)$ of one period. The frequency domain values for this signal $\mathbf{x}$ can be best described as:

    i) finite ii) infinite extent iii) discrete iv) continuous

    a) i) and iii) only b) ii) and iv) only c) ii) only d) ii) and iv) only e) ii) and iii) only

    Correct Answer: ii) and iii) only

20. Let $\mathbf{x}$ be a signal whose frequency values are discrete of infinite extent. Then $\mathbf{x}$ can be represented as:

    a) a constant b) a step function c) a delta function d) a Fourier series e) an impulse response

    Correct Answer: a Fourier series