Test ID: TESTID   Name: ____________________

MAT 321   **Midterm Exam**   Spring 2021

1. Let $\mathbf{x}$ and $\mathbf{y}$ be discrete signals (infinite time extent with values $t \in \mathbb{Z}$) defined by $x_t = t$ for $t \geq 0$, $x_t = 0$ otherwise, and $y_t = e^{i\frac{\pi}{3}t}$, for $-2 \leq t \leq 2$, $y_t = 0$ otherwise. Find the inner product $< \mathbf{x}, \mathbf{y} >$:

   a) $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ b) $\frac{1}{2} - \sqrt{3}i$ c) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$ d) $-\frac{1}{2} - \sqrt{3} - \frac{\sqrt{3}}{2}i$ e) $-\frac{1}{2} + \sqrt{3} + \frac{\sqrt{3}}{2}i$

   Correct Answer: $-\frac{1}{2} - \frac{3}{2}\sqrt{3}i$

2. Same signals $\mathbf{x}$ and $\mathbf{y}$ as in the previous question. Let $\mathbf{w}$ be the convolution $\mathbf{x} * \mathbf{y}$. Find $\mathbf{w}(1)$ or w_1.

   a) $\sqrt{3}i$ b) $2 - \sqrt{3}i$ c) $\sqrt{3} - 2i$ d) $2\sqrt{3} - i$ e) $2\sqrt{3} - i$ $1 - 2\sqrt{3}i$

   Correct Answer: $2 - \sqrt{3}i$

3. Let $\mathbf{x}$ be the same discrete signal as in the previous problem, and now let $\mathbf{y}$ be this signal shifted to the right by 2 samples, so that $y_t = x_{t-2}$. What is the Z-transform of the signal $\mathbf{y}$?

   a) $\frac{z^{-3}}{(1-z^{-1})^2}$ b) $\frac{z^2}{1-z^{-1}}$ c) $\frac{z^{-2}}{1-z^{-1}}$ d) $\frac{1}{1-z^{-3}}$ e) $\frac{1}{(z-1)^3}$

   Correct Answer: $\frac{z^{-3}}{(1-z^{-1})^2}$

4. Let $b(t)$ be the buzz function defined as: $\displaystyle\sum_{k=-\infty}^{+\infty} \delta(t-k)$ where $\delta(t)$ is the 'delta function' which is zero except for $t = 0$ where it has a concentrated area of value one. What is the value of the integral:

$$\int_{-1/2}^{3/2} \sin(\frac{\pi}{3}t)b(t)\ dt$$

   a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) $-\frac{1}{2} - \sqrt{3}$ e) $-\frac{\sqrt{3}}{2}$

   Correct Answer: $\frac{\sqrt{3}}{2}$

5. Solve for a_2 in the following equation:

$$\frac{1}{1 + z^{-1} + 2z^{-2}} = a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots.$$

   a) 2 b) -1 c) 1 d) -2 e) 0

   Correct Answer: -1

6. Let $\mathbf{x}$ be a signal with frequency values $F(\omega)$, and suppose the inverse Fourier Transform of F is computed as:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) e^{i\omega t}\ d\omega.$$

   Then the frequency values ω must be:

   a) continuous, of finite extent b) finite c) discrete, of infinite extent d) continuous, of infinite extent e) none of these

   Correct Answer: continuous, of finite extent

7. Same signal $\mathbf{x}$ as in the previous question. The Z-transform $\mathcal{F}(z)$ can be used to computer $F(\omega)$ as:

   a) $\frac{1}{2\pi}\int_{-\pi}^{\pi} \mathcal{F}(\omega) e^{i\omega t}\ d\omega$ b) $\mathcal{F}(e^{i\omega})$ c) $\frac{1}{\mathcal{F}(e^{i\omega})}$ d) $\int_{-\infty}^{\infty} F(\omega) e^{-i\omega t}\ d\omega$ e) $\mathcal{F}(\omega^2)$

   Correct Answer: $\mathcal{F}(e^{i\omega})$

8. Let $\mathbf{x}$ be a signal which is periodic with continuous time t (finite in extent on an interval of one period). The Inverse Fourier Transform in this context can be thought of as:

a) a Fourier series b) an impulse response c) a delta function d) a phasor e) an integral

Correct Answer: a Fourier series

9. Let $\mathbf{x}$ be the signal $x_t = 1$ for $t = 4k$ or $t = 4k+1$, and $x_t = 0$ for $t = 4k+2$ or $t = 4k+3$, where $k \geq 0$ is any integer, and $x_t = 0$ for $t < 0$. Find the Z-transform of $\mathbf{x}$ by adding two Z-transforms.

a) $\frac{1}{1-z^{-4}}$ b) $\frac{1}{(1-z^{-1})^2}$ c) $\frac{-1}{(z^{-4}-1)}$ d) $\frac{1+z^{-1}}{1-z^{-4}}$ e) $\frac{1}{z(z^3-1)^2}$

Correct Answer: $\frac{1+z^{-1}}{1-z^{-4}}$

10. Simplify: $|1-e^{-i\omega}|^{-2}$

a) $\frac{1}{1-\sin\omega}$ b) $\frac{1}{1+\sin^2\omega}$ c) $\frac{1/2}{1+\cos^2\omega}$ d) $\frac{1/2}{1-\cos\omega}$ e) $\frac{2}{1+\cos^2\omega}$

Correct Answer: $\frac{1/2}{1-\cos\omega}$

11. Let $\mathbf{u}$ be the two-sided phasor signal with values $e^{i\theta t}$, and $\mathbf{v}$ be the one-sided phasor signal with the same values but zero for $t < 0$, and $\mathbf{x}$ be the finite stretch of a phasor, with the same values as $\mathbf{v}$ but zero for $t \geq n$. Also, let $\mathbf{y}$ be the windowed phasor signal which is $\mathbf{x}$ multiplied by the Hamming window. Which signal has the best *resolution* of the frequency θ?

a) $\mathbf{u}$ b) $\mathbf{v}$ c) $\mathbf{x}$ d) $\mathbf{y}$ e) none of them

Correct Answer: $\mathbf{u}$

12. Same signals as in the previous question. Which signals have only finitely many frequencies with non-zero contribution to the spectrum (or frequency content) of the signal?

a) $\mathbf{x}$ and $\mathbf{y}$ only b) $\mathbf{x}$ only c) all of them d) $\mathbf{u}$ only e) none of them

Correct Answer: $\mathbf{u}$ only

13. Let $\mathbf{r}$ be the convolution signal $\mathbf{x} * \mathbf{x}$. If $\mathbf{x}$ has frequency content $X(\omega)$, then $\mathbf{r}$ must have frequency content:

a) $X(\omega)^2$ b) $X(\omega^2)$ c) $X(2\omega)$ d) $2X(\omega)$ e) none of these

Correct Answer: $X(\omega)^2$

14. Same signals as in the previous question. Which signals have frequency content graph which is increasing over some interval of finite extent, say for ω in $[a, b]$, with $a < b$?

a) $\mathbf{u}$ and $\mathbf{v}$ only b) $\mathbf{x}$ only c) $\mathbf{x}$ and $\mathbf{y}$ only d) all but $\mathbf{u}$ e) all of them

Correct Answer: all but $\mathbf{u}$

15. Same signals as in the previous question. The signal $\mathbf{y}$ has the following advantages over the signal $\mathbf{x}$:

i) more nonzero frequency values

ii) better resolution of θ

iii) lower side-lobes

a) ii) and iii) only b) iii) only c) ii) only d) i) only e) i) and iii) only

Correct Answer: ii) and iii) only

16. Consider the linear system:

$$y_t = \sum_{k=-100}^{t} x_k, \; t \geq -100, \qquad y_t = x_t, \; t < -100.$$

Which of the following input signals will have at least one output undefined? Assume the definition holds for all t, unless indicated otherwise.

i) $x_t = (-1)^t$ ii) $x_t = \frac{(-1)^t}{t}, x_0 = 0.$ iii) $x_t = (-1)^t, t \geq 0, \; x_t = 0, t < 0.$

a) i) and ii) only b) none of them c) iii) only d) ii) only e) i) only

Correct Answer: none of them

17. Same system as in the previous problem. Which of the following input signals will have unbounded output (for those outputs which are defined)?

i) $x_t = \frac{(-1)^t}{t}, x_0 = 0$ ii) $x_t = \frac{1}{t}, t > 0, \; x_t = 0, t \leq 0.$ iii) $x_t = \frac{1}{t}, t < 0, \; x_t = 0, t \geq 0.$

a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

Correct Answer: ii) only

18. Find the sum: $\sum_{k=0}^{n-1} e^{i\frac{2\pi}{n-1}k}$, for n a positive integer greater than 1.

a) 1 b) n c) $n - 1$ d) $2\pi n$ e) $\pi/2$

Correct Answer: 1

19. Which of the following signals are absolutely summable? (Recall that this means that the sum of the absolute values of all the signal values converges.) Assume that each signal is zero for $t \leq 0$.

i) $x_t = \sin(\frac{\pi}{2}t)$, ii) $x_t = \frac{1}{t}\sin(\frac{\pi}{2}t)$, iii) $x_t = \frac{1}{t^2}\sin(\frac{\pi}{2}t)$,

a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

Correct Answer: iii) only

20. Which of the following impulse responses will give a system which is BIBO stable? Assume that each signal h_t is zero for $t \leq 0$.

i) $x_t = \cos(\frac{\pi}{2}t)$, ii) $x_t = \frac{1}{t}\cos(\frac{\pi}{2}t)$, iii) $x_t = \frac{1}{t^2}\cos(\frac{\pi}{2}t)$,

a) i) and ii) only b) ii) and iii) only c) iii) only d) ii) only e) i) only

Correct Answer: iii) only