Test ID: TESTID Name: ____________________

MAT 321 **Midterm Exam** Spring 2019

1. Let $b(t)$ be the buzz function defined as: $\sum_{k=-\infty}^{+\infty} \delta(t-k)$ where $\delta(t)$ is the 'delta function' which is zero except for $t = 0$ where it has a concentrated area of value one. What is the value of the integral:

$$\int_{3/2}^{7/2} \cos(\frac{\pi}{4}t)b(t)\ dt$$

   a) $\sqrt{2}/2$ b) $-\sqrt{2}/2$ c) 1 d) -1 e) 0

   Correct Answer: $-\sqrt{2}/2$

2. Let $\mathbf{x}_1$ be the phasor with frequency θ_1 and $\mathbf{x}_2$ be the phasor with frequency θ_2, both discrete signals (infinite time extent with values sampled for $t \in \mathbb{Z}$). If θ_1 and θ_2 are both in the interval $[-\pi, \pi)$ and are not equal, we showed that the inner product of $\mathbf{x}_1$ and $\mathbf{x}_2$ is zero. This came from the fact that an infinite sum of exponentials can be rewritten as a sum of:

   a) delta functions b) Fourier coefficients c) Z-transforms d) rational functions e) polynomials

   Correct Answer: delta functions

3. Let $\mathbf{x}$ be the signal $x_t = 1$ for $t = 4k$ or $t = 4k + 1$, and $x_t = 0$ for $t = 4k + 2$ or $t = 4k + 3$, where $k \geq 0$ is an integer, and $x_t = 0$ for $t < 0$. Find the Z-transform of $\mathbf{x}$ by adding two Z-transforms.

   a) $\frac{1}{1-z^{-4}}$ b) $\frac{1}{(1-z^{-1})^2}$ c) $\frac{-1}{(z^{-4}-1)}$ d) $\frac{1+z^{-1}}{1-z^{-4}}$ e) $\frac{1}{z(z^3-1)^2}$

   Correct Answer: $\frac{1+z^{-1}}{1-z^{-4}}$

4. Let $\mathbf{u}$ be the unit step signal and suppose that $\mathbf{u}$ is fed into a filter which has frequency response function $H(\omega)$. If we know that $H(\frac{\pi}{3}) = 1 + \sqrt{3}i$, then what is the magnitude of the frequency content $|Y(\pi/3)|$ of the output signal $\mathbf{y}$ at $\omega = \pi/3$:

   a) 1 b) 2 c) $\frac{1}{2}$ d) $\sqrt{2}$ e) $\frac{\sqrt{2}}{2}$

   Correct Answer: 2

5. Let $\mathbf{x}_1$ be the infinite extent discrete phasor, $\mathbf{x}_2$ be the one-sided phasor with value 0 for $t < 0$, and let $\mathbf{x}_3$ be the finite stretch phasor, with values 0 for $t < 0$ and $t > n - 1$. Assume that all of these discrete phasor signals have angular frequency θ. Let each of the magnitude frequency content functions for these signals be $|F_1(\omega)|$, $|F_2(\omega)|$, and $|F_3(\omega)|$. Answer the following questions about these functions: (answers are given in order i), ii), iii).)

   i) Which function has the greatest number of zeros for values ω ?

   ii) Which function has the greatest number of changes from increasing to decreasing?

   iii) Which function has best resolution of its frequency content?

   a) $|F_1|, |F_2|, |F_3|$ b) $|F_2|, |F_1|, |F_2|$ c) $|F_3|, |F_2|, |F_1|$ d) $|F_1|, |F_3|, |F_1|$ e) $|F_2|, |F_3|, |F_1|$

   Correct Answer: $|F_1|, |F_3|, |F_1|$

6. Same three signals as in the previous question. Which signals have Z-transform a rational function of z (a quotient of two polynomials) ?

   a) $\mathbf{x}_1$ only b) $\mathbf{x}_1$ and $\mathbf{x}_2$ only c) $\mathbf{x}_2$ and $\mathbf{x}_3$ only d) $\mathbf{x}_1$ and $\mathbf{x}_3$ only e) $\mathbf{x}_2$ only

   Correct Answer: $\mathbf{x}_2$ and $\mathbf{x}_3$ only

7. Let $\mathbf{x}$ and $\mathbf{y}$ be discrete signals (infinite time extent with values $t \in \mathbb{Z}$) defined by $x_t = t$ for $t \geq 0$, $x_t = 0$ otherwise, and $y_t = e^{i\frac{\pi}{4}t}$, for $-2 \leq t \leq 2$, $y_t = 0$ otherwise. Find the inner product $< \mathbf{x}, \mathbf{y} >$:

   a) $\frac{\sqrt{2}}{2} - (\frac{\sqrt{2}}{2} + 2)i$ b) $-\frac{1}{2} - \frac{\sqrt{2}}{2}i$ c) $-\frac{\sqrt{2}}{2} + (\frac{\sqrt{2}}{2} + 2)i$ d) $\frac{\sqrt{2}}{2} - 2 + (\frac{\sqrt{2}}{2} + 2)i$ e) $\frac{\sqrt{2}}{2} - 2 + \frac{\sqrt{2}}{2}i$

   Correct Answer: $\frac{\sqrt{2}}{2} - (\frac{\sqrt{2}}{2} + 2)i$

8. Same signals $\mathbf{x}$ and $\mathbf{y}$ as in the previous question. Let $\mathbf{w}$ be the convolution $\mathbf{x} * \mathbf{y}$. Find $\mathbf{w}(1)$ or w_1.

   a) $1 - \sqrt{2} + (2 - \sqrt{3})i$ b) $-\frac{\sqrt{2}}{2} - \sqrt{3}i$ c) $1 + \sqrt{2} - (\sqrt{2} + 3)i$ d) $-\frac{1}{2} + \frac{3}{2}\sqrt{2}i$ e) $\frac{1}{2} + \frac{5}{2}\sqrt{3}i$

   Correct Answer: $1 + \sqrt{2} - (\sqrt{2} + 3)i$

9. Identify the series $g(z) = 1 + 2z^{-2} + 3z^{-4} + 4z^{-6} - \cdots$ as $-\frac{1}{2}z^3 f'(z)$, for a simple series $f(z)$. Find the closed form of $g(z)$. (Alternatively, you can find the series for $g(z)$ as a product of two geometric series whose closed forms are known.)

   a) $\frac{1}{1-z^{-1}}$ b) $\frac{1}{(1-z^{-1})^2}$ c) $\frac{-1}{(z-1)}$ d) $\frac{1}{(1-z^{-2})^2}$ e) $\frac{1}{z(z-1)^2}$

   Correct Answer: $\frac{1}{(1-z^{-2})^2}$

10. Let $\mathbf{x}$ be the discrete signal with Z-transform $g(z)$ as in the previous problem, and now let $\mathbf{y}$ be this signal shifted to the right by 4 samples, so that $y_t = x_{t-4}$. What is the Z-transform of the signal $\mathbf{y}$?

    a) $\frac{1}{(z^2-1)^2}$ b) $\frac{z^{-1}}{1-z^{-2}}$ c) $\frac{z^2}{1-z^{-2}}$ d) $\frac{z^{-2}}{(1-z^{-1})}$ e) $\frac{1}{1-z^{-2}}$

    Correct Answer: $\frac{1}{(z^2-1)^2}$

11. Solve for a_2 in the following equation:

$$\frac{1}{1 - z^{-1} + 3z^{-2}} = a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots.$$

    a) 1 b) -1 c) -2 d) 2 e) 0

    Correct Answer: -2

12. Simplify: $|1 - e^{-i\omega}|^{-2}$

    a) $\frac{1/2}{1-\cos\omega}$ b) $\frac{1}{1-\sin\omega}$ c) $\frac{1}{1+\sin^2\omega}$ d) $\frac{1/2}{1+\cos^2\omega}$ e) $\frac{2}{1+\cos^2\omega}$

    Correct Answer: $\frac{1/2}{1-\cos\omega}$

13. Fill in the blanks in this sentence: "The _______________ of a filter is the same as the Z-transform of the filter's _______________."

    a) transfer function, impulse response b) transfer function, frequency response c) impulse response, transfer function d) frequency response, transfer function e) frequency response, impulse response

    Correct Answer: transfer function, impulse response

14. Suppose a raised cosine window is given as $h_t = B - A\cos(2\pi t)$ on the interval $[0, 1]$. Solve for A and B so that this window has a maximum value of 1 and a minimum value of 0.1 on the interval $[0, 1]$. What is A?

    a) 0.35 b) 0.45 c) 0.55 d) 0.40 e) 0.50

    Correct Answer: 0.45

15. Suppose that a function ζ takes signals to their Z-transforms, so that for instance if $\mathbf{u}$ is the unit step signal then

$$\zeta(\mathbf{u}) = \frac{1}{1 - z^{-1}}.$$

    If $\mathbf{x}$ and $\mathbf{y}$ are two signals, then what is $\zeta(\mathbf{x} * \mathbf{y})$?

    a) $\zeta(\mathbf{x})\zeta(\mathbf{y})$ b) $\zeta(\mathbf{x}) * \zeta(\mathbf{y})$ c) $\zeta(\mathbf{x})/\zeta(\mathbf{y})$ d) $\zeta(\mathbf{x}) + \zeta(\mathbf{y})$ e) $\frac{1}{1-\zeta(\mathbf{x})\zeta(\mathbf{y})}$

    Correct Answer: $\zeta(\mathbf{x})\zeta(\mathbf{y})$