Math 351 Syllabus

Semester:	Summer 2019
Title:	Quaternions, Interpolation, and Animation
Times:	WF 2:00 - 3:40
Place:	Jimbo
Instructor:	Dr. Matt Klassen
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Phone:	(425) 895-4423
Office hours:	W-F 12:00-1:00 in Office, or by appointment
Course Home Page:	http://azrael.digipen.edu/MAT351

WEB PAGES AND MOODLE:

The Moodle page for MAT351 will contain a link to the course web page. The web page is the central repository for all course documents, including homework assignments. Updates to homework will be posted on the web page only. Scores for quizzes, homework, exams, and projects, will be posted through perl scripts on the course web page.

The Moodle page will be primarily used for chat, forums, and for submission of projects.

MATERIALS:

Text: None

Class Notes: Material on Groups, Fields, Vector spaces, Algebras, Quaternion properties, differential geometry of curves on the quaternion sphere, splines, interpolation techniques.

Reference Materials:

1. Koecher, Remmert: "Complex Numbers."
2. Koecher, Remmert: "Real Division Algebras."
3. Ken Shoemake: "Animating Rotation with Quaternion Curves."
4. Watt and Watt: "Advanced Animation and Rendering Techniques"
5. Kim et al: "A Compact Differential Formula for the First Derivative of a Unit Quaternion Curve."
6. Barr et al : "Smooth Interpolation of Orientations with Angular Velocity Constraints using Quaternions."
7. Barr et al : "Fast Construction of Accurate Quaternion Splines."
8. Barr et al : "Dynamic Splines with Constraints for Animation."

BACKGROUND MATHEMATICS:

Linear Algebra, and geometry of curves.

COURSE GOALS AND OBJECTIVES:

1) Students will learn the concepts of abstract algebra, especially as they apply to quaternions and the animation of rotations. They will demonstrate their understanding through a programming project and also through various homework and quizzes.

2) Students will become familiar with basic concepts of objects such as groups, rings, fields, and algebras.

3) Students will solidify their knowledge of Linear Algebra, and geometry of curves and vectors, by using it as a tool to solve problems involving the interpolation of orientations in three and four dimensions.

QUIZZES AND EXAMS:

Periodic quizzes will be given to test understanding of the basic lecture material, including definitions and examples. There will be a midterm exam and a final exam.

GRADING:

Midterm Exam	20%
Final	20%
Homework	20%
Quizzes	20%
Project	20%

Grades will be determined based on total course percentage. Percentage scores will determine letter grades according to the scale: (in the worst case)

A	$93 - 100$
A-	$90 - 92.9$
B+	$87 - 89.9$
B	$83 - 86.9$
B-	$80 - 82.9$
C+	$77 - 79.9$
C	$73 - 76.9$
C-	$70 - 72.9$
D	$60 - 69.9$
F	< 60

ACADEMIC INTEGRITY POLICY:

Academic dishonesty, or cheating, occurs when a student represents someone else's work as their own. This can occur on exams, quizzes, homework, or projects. The DigiPen catalog states that such activity will not be tolerated. Further guidelines are given in the handout on DigiPen's Academic Integrity Policy. Please read these carefully and help us to promote an environment of academic integrity where all students are encouraged to learn to the best of their ability.

COURSE DESCRIPTION:

This course gives an introduction to several mathematical topics of foundational importance to abstract algebra, and in particular the algebra of quaternions. Topics covered may include: operations, groups, rings, fields, vector spaces, algebras, complex numbers, quaternions, curves over the quaternionic space, interpolation techniques, splines, octonions, and Clifford algebras.

HOMEWORK ASSIGNMENTS:

Homework will be assigned and posted on the web page and collected approximately weekly. All homework should be submitted to the collection box in the front office before it closes on the due date. Students are expected to ask for clarification and/or assistance on homework during office hours. The homework assignments are a key part of the course and are expected to be done carefully and honestly. Students are encouraged to work together when discussing homework, but are reminded that any homework turned in for a grade must be written up entirely by that student, and not copied from another's work.

PROJECT:

1. First phase: (static orientation display) Use 3D graphics engine such as DirectX, OpenGL, or Swift/SceneKit, to render a model in 3D from a fixed view point. The model should be complex enough to exhibit well any slight changes in orientation through reflections in lighting. (A cube is too simple.) The program should then accept a list of orientations (given as axis-angle pairs) as a text file, and provide an interface to step through these orientations as applied to the model, keeping the view point fixed.

2. Second phase: (text only interpolation) In this phase the text file of orientations should be converted to quaternions and intermediate quaternions filled in according to several different techniques. The first will be Slerp, and the second will be a smooth interpolation called Shoemake Bezier. In order to see the difference numerically, another text file will be created with the successive differences in slopes of lines through intermediate quaternions.

3. Third phase: (graphical interpolation) In this phase the first and second parts are extended by filling in the graphical images that correspond to the intermediate orientations. This can be accomplished by many different interpolation techniques. The simplest is called Slerp (for spherical linear interpolation). The first smooth interpolation is called Shoemake Bezier. We will also use Spherical Quadrangle and Circular Blending techniques. Other techniques will be discussed in class. The program should implement at least three of these techniques and have an interface that allows the user to select the technique.

TENTATIVE WEEKLY TOPICS:

Week	Dates	Topics
1	May 8, 10	Rotations, orientations, and overview of project
2	May 15, 17	Groups: axioms, permutations, matrix groups
3	May 22, 24	Finite quaternion group, rings and fields
4	May 29, 31	Real Algebras, basis representation
5	June 5, 7	Cross product and matrix algebras
6	June 12, 14	Complex numbers, subspaces and transformations
7	June 19, 21	Midterm Exam: Wednesday, June 19
8	June 26, 28	orthogonal groups, parametrizations of rotations
9	July 3, 5	Hamiltons quaternions, basic properties, subspaces and subgroups
10	July 10, 12	Reflections, Hamilton and Cayley Theorems, Interpolation techniques: slerp, quadrangle, circular blending
11	July 17, 19	Unit quaternion sphere, covering groups
12	July 24, 26	Final Exams