

## MAT 351

## Quiz 1

## Summer 2019

Recall: The order of an element  $x$  in a group  $G$  is the smallest positive integer  $k$  such that  $x^k = e$ , where  $e$  is the identity of  $G$  and the group operation  $*$  has been interpreted as multiplication.

1. Let  $G$  be a group with elements  $a, b, c \in G$ , with identity  $e$ . (Assume the binary operation is multiplication.) Which of the following equations must always hold in  $G$ ?

i)  $(ab)^{-1} = b^{-1}a^{-1}$       ii)  $(ab)^2 = a^2b^2$       iii)  $(ab)^2 = a(ba)b$

- a) ii) only      b) i) only      c) i) and ii) only      d) ii) and iii) only      e) i) and iii) only

Correct Answer: i) and iii) only

2. How many elements of order 2 are there in the group  $\mathbb{Z}_6$ ?

- a) 2      b) 1      c) 3      d) 4      e) 0

Correct Answer: 1

3. How many elements of order 3 are there in the quaternion group  $Q_8$ ?

- a) 1      b) 2      c) 3      d) 4      e) 0

Correct Answer: 0

4. How many elements of order 2 are there in  $\mathbb{Z}_2 \times \mathbb{Z}_2$ ?

- a) 3      b) 2      c) 0      d) 1      e) 4

Correct Answer: 3

5. What is the order of the matrix  $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$  in the group  $GL_2(\mathbb{F}_2)$ ? (The group operation is matrix multiplication, where both addition and multiplication of matrix entries are done mod 2.)

- a) 3      b) 2      c) 0      d) 1      e) 4

Correct Answer: 3

6. Same matrix  $A$  as in the previous problem. Find the inverse of  $A$ . What is  $A + A^{-1}$ ?

- a)  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$       b)  $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$       c)  $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$       d)  $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$       e)  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Correct Answer:  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

7. What is the order of the complex number  $\frac{1}{2} + \frac{\sqrt{3}}{2}i$  when using complex multiplication as the group operation?

- a) 1      b) 2      c) 3      d) 4      e) 6

Correct Answer: 6

8. Let  $R$  be a  $3 \times 3$  rotation matrix with axis vector  $\mathbf{u}$  and angle  $\theta$ . Let  $B$  be the matrix defined as  $B = R + R^T + (1 - \text{tr}(R))I$ . Also let  $\mathbf{x}$  be an arbitrary vector in  $\mathbb{R}^3$ , and let  $k$  represent any real constant. The matrix  $B$  satisfies which of the following properties:

i)  $B$  is symmetric      ii)  $B\mathbf{x} = k\mathbf{u}$  for some  $k$       iii)  $B = R^{-1}$

- a) ii) only      b) i) only      c) i) and ii) only      d) ii) and iii) only      e) i) and iii) only

Correct Answer: i) and ii) only

9. Find the product of the two permutations of the set  $\{1, 2, 3\}$  given as  $a$  and  $b$  if  $a$  swaps 1 and 2, and  $b$  swaps 2 and 3. Write the product  $ab$  using function notation, so that  $b$  acts first. What is the cycle notation for  $ab$ ?

- a) (123)      b) (213)      c) (13)      d) (12)      e)  $e$

Correct Answer: (123)

10. Same  $a$  as in previous question but now suppose that  $b$  is given in cycle notation as (132). What is the cycle notation for  $ab$ ?

- a) (123)      b) (213)      c) (13)      d) (12)      e)  $e$

Correct Answer: (13)