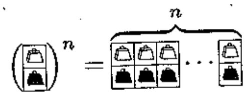
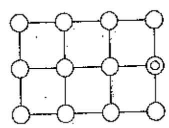
1. Consider the $2 \times n$ CLOBBER position



Show that if n is even then

is a second-player win. (By the way, the first player wins when $n \leq 13$ is odd and, we conjecture, for all n odd.)

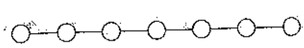
2. Prove that Left to move can win in the COL position



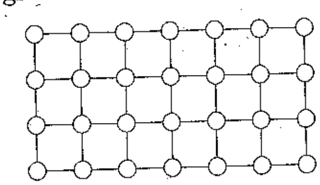
3. Suppose two players play STRINGS & COINS with the additional rule that a player, on her turn, can spend a coin to end her turn. The last player to play wins. (Spending a coin means discarding a coin that she has won earlier in the game.)
   (a) Prove that the first player to take any coin wins.
   (b) Suppose the players play on an m-coin by n-coin board with the usual starting position. Prove that if $m + n$ is even, the second player can guarantee a win.
   (c) Prove that if $m + n$ is odd, the first player can guarantee a win.

4. Two players play the following game on a round tabletop of radius R. Players take turns placing pennies (of unit radius) on the tabletop, but no penny is allowed to touch another or to project beyond the edge of the table. The first player who cannot legally play loses. Determine who should win as a function of R.[5]

5. Who wins SNORT when played on a path of length n?



How about an $m \times n$ grid?



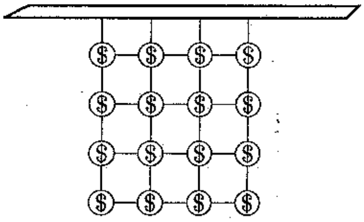
[5]The players are assumed to have perfect fine motor control!

6. The game of ADD-TO-15 is the same as 3-TO-15 (page 16) except that the first player to get *any* number of cards adding to 15 wins. Under perfect play, is ADD-TO-15 a first-player win, second-player win, or draw?

7. The following vertex deletion game is played on a directed graph. A player's turn consists of removing any single vertex with even indegree (and any edges into or out of that vertex.) Determine the winner if the start position is a directed tree, with all edges pointing toward the root.

8. Two players play a vertex deletion game on an undirected graph. A turn consists of removing exactly one vertex of even degree (and all edges incident to it.) Determine the winner.

9. A bunch of coins is dangling from the ceiling. The coins are tied to one another and to the ceiling by strings as pictured below. Players alternately cut strings, and a player whose cut causes any coins to drop to the ground loses. If both players play well, who wins?



10. The game of SPROUTLETTES is played like SPROUTS, only the maximum allowed degree of a node is only 2. Who wins in SPROUTLETTES?

11. Find a winning strategy for BRUSSELS SPROUTS. (*Hint*: Describe how end positions must look and deduce how many moves the game lasts.)

12. How many moves does a game of SPROUTS last as a function of both the number of initial dots and the number of isolated degree two nodes at the end of the game? Give a rule for playing SPROUTS analogous to the number of long chains in DOTS & BOXES.

13. Prove that the first player wins at HEX. You are free to find and present a proof you find in the literature, but be sure to cite your source and rephrase the argument in your own words.

14. SQUEX is a game like HEX but is played on a square board. A player makes a turn by placing a checker of her own color on the board. Squares on the board are *adjacent* if they share a side. Black's goal is to connect the top and bottom edges with a path of black checkers, while White wishes to connect the left and right edges with white checkers.