

MAT 364/564 Final Exam Answer Sheet

Fall 2020

Quiz ID: ZFM

Name: \_\_\_\_\_

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## MAT 364/564

## Final Exam

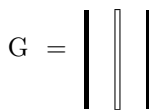
Fall 2020

- For True-False questions, a statement is only True if it must always be True under the given assumptions, otherwise it is False.
- If a game  $G$  is *not* a combinatorial game, we will say that it does not have outcome class given by any one of  $\mathcal{N}$ ,  $\mathcal{P}$ ,  $\mathcal{R}$  or  $\mathcal{L}$ . So the outcome class of  $G$  is “none of the above”.
- A losing move is one which moves into a position in which the other player can force a win playing first.
- The symbol  $\oplus_n$  stands for the game “Tiny- $n$ ”.
- $G \cong H$  means  $G$  is isomorphic to  $H$
- $G = H$  means  $G$  is equal (as games) to  $H$
- $OC(G)$  means Outcome Class of  $G$
- unless otherwise stated, games are two-player finite combinatorial games

1. Which of the following are True implications for any two games  $G$  and  $H$ ?

- i)  $G \cong H \implies G = H$       ii)  $G = H \implies G \cong H$       iii)  $OC(G) = OC(H) \implies G = H$   
 a) iii) only      b) ii) and iii) only      c) ii) only      d) i) only      e) i) and iii) only

2. In each of the Toppling Dominoes and Domineering games  $G$  and  $H$  below, there is a reversible option on the Left side of the game tree. What is this reversible option equal to?



$H =$ 

X		X
		X
X		
X		X

- a)  $*$       b)  $0$       c)  $\frac{1}{2}$       d)  $1$       e)  $-1$

3. Choose a True statement about the Toppling Dominoes and Domineering games  $G$  and  $H$  from the previous question.

- a)  $G = 0$       b)  $G = *$       c)  $H = \frac{1}{2}$       d)  $G = \frac{1}{2}$       e)  $H = 0$

4. Choose the best inequality or other symbol for the Toppling Dominoes and Domineering games  $G$  and  $H$  from the previous question.

- a)  $G < H$       b)  $G = H$       c) none      d)  $G > H$       e)  $G || H$

5. Suppose  $G$  is a combinatorial game in which all of Left's options from  $G$  are in  $\mathcal{N}$ , and all of Right's options from  $G$  are in  $\mathcal{L}$ . What can be said about the outcome class of  $G$ ?
- a)  $G$  is in  $\mathcal{P}$       b) none of the above      c)  $G$  is in  $\mathcal{N}$       d)  $G$  is in  $\mathcal{R}$  or  $\mathcal{P}$       e)  $G$  is in  $\mathcal{N}$  or  $\mathcal{P}$
6. A game  $G$  of Nim is played with two stacks of two counters each, three stacks of three counters each, and one stack of four counters. What is the outcome class of this game?
- a)  $\mathcal{N}$       b) none of the above      c)  $\mathcal{R}$       d)  $\mathcal{L}$       e)  $\mathcal{P}$
7. The game of domineering is played on two  $2 \times 5$  rectangular boards, as a sum of two games. (Each board has two rows and five columns.) What is the outcome class of this game?
- a)  $\mathcal{N}$       b) none      c)  $\mathcal{R}$       d)  $\mathcal{L}$       e)  $\mathcal{P}$
8. Same game as in the previous question. What is the height of the game tree?
- a) 11      b) 8      c) 10      d) 9      e) 7
9. Same game as in the previous question. If the game is played at random, what is the maximum number of moves that the game will last (considering that Right or Left may play first)?
- a) 6      b) 8      c) 9      d) 10      e) 7

10. Suppose that a combinatorial game  $G$  has the property that three of Left's options from  $G$  are in outcome class  $\mathcal{R}$ , and two of Left's options from  $G$  are in outcome class  $\mathcal{N}$ . Suppose also that  $L$  has a total of 6 options from  $G$  and that  $R$  has a total of 6 options from  $G$ . What can you say about the outcome class of  $G$ ?
- a)  $G \in \mathcal{L}$       b)  $G \in \mathcal{N} \cup \mathcal{L}$       c)  $G \in \mathcal{P} \cup \mathcal{N}$       d)  $G \in \mathcal{P}$       e) none of the above
11. Let  $G = \{G_1^L, G_2^L | G_1^R, G_2^R\}$ , with  $G_1^L = \{H_1 | H_2\}$ ,  $G_2^L = \{H_3 | H_4\}$ ,  $G_1^R = \{H_5 | H_6\}$ ,  $G_2^R = \{H_7 | H_8\}$ , and  $H_1 \in \mathcal{N}$ ,  $H_2 \in \mathcal{P}$ ,  $H_3 \in \mathcal{L}$ ,  $H_4 \in \mathcal{N}$ ,  $H_5 \in \mathcal{P}$ ,  $H_6 \in \mathcal{N}$ ,  $H_7 \in \mathcal{P}$ , and  $H_8 \in \mathcal{P}$ . What can you say about the outcome class of  $G$ ?
- a)  $G \in \mathcal{R}$       b)  $G \in \mathcal{P}$       c)  $G \in \mathcal{L}$       d)  $G \in \mathcal{N}$       e) none of the above
12. Let  $G$  be the 3-player game of Nim with *three* stacks having 1 counter each, and *one* stack have two counters. Assume that a player loses if they cannot move, and a player wins if both of the other players lose. Determine which of the following statements are True:
- i) Only Player 1 has a winning strategy  
 ii) Only Player 2 has a winning strategy  
 iii) Only Player 3 has a winning strategy  
 iv) None of the players has a winning strategy
- a) iii) is True      b) none of them are True      c) ii) is True      d) i) is True      e) iv) is True
13. Let  $G$  be the sum of two  $3 \times 3$  domineering square games. What is the outcome class of  $G$ ?
- a) none of the above      b)  $G \in \mathcal{R}$       c)  $G \in \mathcal{L}$       d)  $G \in \mathcal{P}$       e)  $G \in \mathcal{N}$
14. Same  $G$  as in the previous question. What is the height of game tree of  $G$ ?
- a) 7      b) 4      c) 5      d) 3      e) 8
15. Let  $G$  be the  $2 \times 4$  domineering game. Given that Right plays first, and both players play randomly, what is the probability Left will win? (Hint: consider the two types of Right first moves up to symmetry, one with probability  $2/3$  and one with probability  $1/3$ . Then determine what fraction of the time Left will win in each of those cases. Keep in mind that whenever a player has more than one option, each one is equally likely.)
- a)  $\frac{1}{6}$       b)  $\frac{2}{3}$       c)  $\frac{1}{4}$       d)  $\frac{1}{3}$       e)  $\frac{1}{2}$

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16. Which of the following are True statements?

i) The game  $\pm 1$  is all-small

ii) The game  $\oplus_1$  (Tiny<sub>1</sub>) is all-small

iii) Some all-small games are partizan

a) FFT

b) FFF

c) TTF

d) TFF

e) TFT

17. Determine the value of the Amazons game below:

○			X
X			●

a)  $2 \pm 1$

b)  $-\frac{3}{2}$

c)  $*2$

d)  $2 + *$

e)  $\pm 3$

18. Determine the value of the Amazons game below:

	●		○			○
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a)  $-1 \pm 1$

b)  $-\frac{1}{2}$

c)  $*$

d)  $-1 + *$

e)  $\pm 2$

19. Determine the value of the Amazons game below:

	X		○
	●		X

a)  $2 \pm 1$

b)  $\frac{3}{2}$

c)  $*$

d)  $2 + *$

e)  $\pm 3$

20. Let  $H$  be the game:  $\{\{1|-1\} \parallel \{0|-1\}\}$ , and let  $K$  be the game:  $\{\{2|-2\} \parallel \{3|1\}\}$ . Define  $G$  to be the game:

$$\{-2, \{2|-1\}, H \parallel \{3|-3\}, K\}$$

What is  $LS(G)$ , the left stop of  $G$ ?

- a)  $-1$                       b)  $0$                       c)  $2$                       d)  $1$                       e)  $-2$

21. Same game as in the previous problem. Find  $RS(G)$ :

- a)  $-2$                       b)  $0$                       c)  $2$                       d)  $1$                       e)  $-1$

22. What is the game tree height of the game  $\frac{39}{32}$  ?

- a)  $6$                       b)  $5$                       c)  $9$                       d)  $8$                       e)  $7$

23. What is the maximum number of moves that can be played in the game  $\frac{39}{32}$  ?

- a)  $2$                       b)  $5$                       c)  $3$                       d)  $4$                       e)  $6$

24. What is the value of the game  $\{\frac{19}{16} \mid \frac{39}{32}\}$  ?

- a)  $\frac{21}{16}$                       b)  $\frac{37}{32}$                       c)  $\frac{9}{8}$                       d)  $\frac{5}{4}$                       e)  $\frac{77}{64}$

25. Which of the following games  $G$  satisfies:  $G < \pm 1$ ?

- i)  $*$                       ii)  $-1$                       iii)  $-2$   
a) iii) only                      b) ii) and iii) only                      c) i) only                      d) ii) only                      e) i) and iii) only

26. Let  $H$  be the  $2 \times 4$  domineering rectangle game. ( $H$  has 2 rows and 4 columns, Left plays vertical and Right plays horizontal.) What is the outcome class of  $H$ ?
- a)  $\mathcal{N}$                       b)  $\mathcal{R}$                       c) none of these                      d)  $\mathcal{L}$                       e)  $\mathcal{P}$
27. Same game  $H$  as in the previous question. Draw the game tree of  $H$ . (Do not include redundant or isomorphic branches.) Before doing any (dominated or reversible) pruning, how many dominated options are there on the left side of the game tree?
- a) 2                      b) 4                      c) 1                      d) 0                      e) 3
28. Same game  $H$  and tree as in the previous question. Before doing any (dominated or reversible) pruning, how many reversible options are there on the left side of the game tree?
- a) 2                      b) 4                      c) 1                      d) 0                      e) 3
29. Now switch to the right side. Same game  $H$  and tree as in the previous question. Before doing any (dominated or reversible) pruning, how many dominated options are there on the *right* side of the game tree?
- a) 2                      b) 4                      c) 1                      d) 0                      e) 3
30. Same game  $H$  and tree as in the previous question. Find the canonical form of  $H$ :
- a)  $\{2|0 \parallel 0|-2\}$     b)  $\{-\frac{1}{2}|* \parallel 2|0\}$     c)  $\{0|-2 \parallel 2|-\frac{1}{2}\}$     d)  $\{2|-\frac{1}{2} \parallel 0|-2\}$     e)  $\{2|0 \parallel -\frac{1}{2}|*\}$