

MAT 364/564

Midterm Exam

Fall 2018

1. Suppose G is a combinatorial game and that one of Left's options from G is to a \mathcal{P} position, and one of Right's options from G is to an \mathcal{L} position. Given this information, what is the most accurate description of the outcome class of G ?

a) \mathcal{N} b) $\mathcal{N} \cup \mathcal{L}$ c) \mathcal{P} d) \mathcal{L} e) $\mathcal{P} \cup \mathcal{L}$

Correct Answer: $\mathcal{N} \cup \mathcal{L}$

2. Determine the outcome class of the Amazon's game G below:

X		•
◦		X

a) \mathcal{P} b) \mathcal{L} c) \mathcal{R} d) \mathcal{N} e) none of the above

Correct Answer: \mathcal{N}

3. Same G as in the previous question. What is the birthday (height of game tree) of G ?

a) 5 b) 7 c) 4 d) 6 e) 8

Correct Answer: 5

4. Let G be the sum of toppling dominoes games $1010 + 10$ where 1 denotes a black (Left) and 0 denotes a white (Right) domino.

Determine the outcome class of G :

a) \mathcal{P} b) \mathcal{L} c) \mathcal{R} d) \mathcal{N} e) none of the above

Correct Answer: \mathcal{N}

5. Same game G as in the previous question. What is the height of the game tree?

a) 5 b) 7 c) 4 d) 6 e) 4

Correct Answer: 6

6. Let G be the greedy Nim game with stacks of counters of heights: 4, 3 and 2. If first player takes 2 counters, how many counters should second player take?

a) 5 b) 2 c) 3 d) 1 e) 4

Correct Answer: 3

7. The game of domineering is played on the board below. What is the outcome class of this game?

		X	X
X			X

- a) \mathcal{L} b) \mathcal{R} c) \mathcal{N} d) \mathcal{P} e) none

Correct Answer: \mathcal{R}

8. Same game as in the previous question. What is the height of the game tree?

- a) 5 b) 4 c) 6 d) 7 e) 8

Correct Answer: 4

9. Same game as in the previous question. If the game is played at random, what is the maximum number of moves that the game will last?

- a) 5 b) 6 c) 7 d) 4 e) 8

Correct Answer: 4

10. Same game as in the previous question. If Left plays first, how many winning moves does Left have?

- a) 0 b) 1 c) 3 d) 2 e) 4

Correct Answer: 0

11. Same game as in the previous question. If Right plays first, how many winning moves does Right have?

- a) 0 b) 1 c) 3 d) 2 e) 4

Correct Answer: 2

12. Assume that a property X of games is verified to be *false* for games of height 0, 2, 4, 6, 8 and 10, but *true* for games of height 11. It is then also shown that if property X is assumed to be true for all games of height at most $n - 1$, then property X must be true for all games of height n . Now suppose that G is a particular game of height 12. In addition to the above, what do we need to do in order to prove that G has property X ?

- a) verify property X for games of height $n = 12$ b) verify property X for games of height $n = 1$ c) verify property X for games of height $n \leq 9$ d) verify property X for games of height $n = 9$ e) nothing

Correct Answer: verify property X for games of height $n = 12$ OR nothing

13. Let Y be the property that a game is partizan. (A game is partizan if the options are not always the same for both players at any node of the game tree.) Which of the following fails to be true, or fails to be true by assumption, or fails to be provable, in attempting to prove by induction on game tree height that all games are partizan? Assume that the base case refers to height zero.

- i) the base case ii) the induction step iii) the induction hypothesis
a) i) only b) ii) only c) iii) only d) i) and ii) only e) none of the above

Correct Answer: i) only

14. Let Y be the property that a game is in Lwpcf or Rwpf. Which of the following fails to be true, or fails to be true by assumption, or fails to be provable, in attempting to prove by induction on game tree height that all games are in Lwpcf or Rwpf? Assume that the base case refers to height zero.

- i) the base case ii) the induction step iii) the induction hypothesis
a) i) only b) ii) only c) iii) only d) i) and ii) only e) none of the above

Correct Answer: i) and ii) only

15. How many *partizan* games (up to isomorphism) are there with height *at most* one?

- a) 0 b) 1 c) 2 d) 3 e) 4

Correct Answer: 2

16. Suppose G and H are combinatorial games with G in \mathcal{L} and H in \mathcal{P} . What are the *possible* outcomes classes of the game $G + H$?

- a) \mathcal{L} , \mathcal{N} or \mathcal{P} only b) \mathcal{L} or \mathcal{P} only c) \mathcal{N} , \mathcal{L} , \mathcal{R} or \mathcal{P} d) \mathcal{L} only e) \mathcal{N} or \mathcal{L} only

Correct Answer: \mathcal{L} only

17. Suppose G and H are combinatorial games with G in \mathcal{N} and H in \mathcal{N} . What are the *possible* outcomes classes of the game $G + H$?

- a) \mathcal{L} or \mathcal{R} only b) \mathcal{N} or \mathcal{P} only c) \mathcal{N} , \mathcal{L} , \mathcal{R} or \mathcal{P} d) \mathcal{P} only e) \mathcal{N} only

Correct Answer: \mathcal{N} , \mathcal{L} , \mathcal{R} or \mathcal{P}

18. Suppose H is the 2×7 domineering rectangle. What is the outcome class of H ?

- a) \mathcal{P} b) \mathcal{R} c) \mathcal{L} d) \mathcal{N} e) none

Correct Answer: \mathcal{N}

19. Suppose H is the 2×7 domineering rectangle. What is the height of the game tree?

- a) 5 b) 7 c) 6 d) 3 e) 4

Correct Answer: 7

20. Suppose H is the 2×7 domineering rectangle. Suppose that Right's hand is tied by not allowing Right to play moves that cross over columns 4 and 5. Using the One Hand Tied Principle, what conclusion can be made about the 2×7 rectangle?

- a) in Rwpf b) in Rwps c) in Lwpf d) in Lwps e) no conclusion

Correct Answer: in Rwpf