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MAT 364/564 **Midterm Exam** Fall 2020

1. Suppose G is a combinatorial game and that one of Left's options from G is to an $\mathcal{R}$ position, and one of Right's options from G is to a $\mathcal{P}$ position. Given this information, (and making no other assumptions about the game tree), what is the most accurate description of the outcome class of G?

   a) $\mathcal{L}$ b) $\mathcal{N} \cup \mathcal{R}$ c) $\mathcal{P}$ d) $\mathcal{R}$ e) $\mathcal{P} \cup \mathcal{L}$

   Correct Answer: $\mathcal{N} \cup \mathcal{R}$

   Note: L has a losing move playing first (no info). R has a winning move playing first (so Rwpf = R union N)

2. Determine the outcome class of the Amazon's game G below. Black is Left and White is Right.

X	X	
	•	
○		X

   a) $\mathcal{P}$ b) $\mathcal{L}$ c) $\mathcal{R}$ d) $\mathcal{N}$ e) none of the above

   Correct Answer: $\mathcal{L}$

   Note: Left wins easily playing first, by moving down and shooting up. Also, Left can respond to any Right move by playing to a $\mathcal{P}$ position.

3. Same G as in the previous question. What is height of game tree of G?

   a) 5 b) 7 c) 4 d) 6 e) 8

   Correct Answer: 4

4. Let G be the sum of toppling dominoes games $1010 + 10$ where 1 denotes a black (Left) and 0 denotes a white (Right) domino. Determine the outcome class of G:

   a) $\mathcal{P}$ b) $\mathcal{L}$ c) $\mathcal{R}$ d) $\mathcal{N}$ e) none of the above

   Correct Answer: $\mathcal{N}$

   Either player can win by first toppling the left game to be equal to the right.

5. Same game G as in the previous question. What is the height of the game tree?

   a) 5 b) 7 c) 4 d) 6 e) 4

   Correct Answer: 6

   Note: there are six very bad moves, each toppling one domino to the left.

6. Let G be the *Greedy Nim* game with exactly three stacks of counters of heights: 4, 3 and 2. If first player takes 2 counters, how many counters should second player take to win? (Answers of type "a or b" imply that there are two ways to win, by taking either a or b.)

   a) 1 or 3 b) 1 or 2 c) 2 or 3 d) 2 only e) 3 only

   Correct Answer: 2 or 3

   Note: Player always take from a largest stack. After the first move the stacks are 2, 3 and 2. Second player should move to an even number of highest stacks to win by taking 2 or 3 from the middle stack.

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7. The game of domineering is played on the board below. If Right plays first, how many winning moves does Right have?

		X	
X			X

a) 0 b) 1 c) 3 d) 2 e) 4

Correct Answer: 2

Note: Right wins by starting in the upper left, or middle left.

8. Same game as in the previous question. If Left plays first, how many winning moves does Left have?

a) 0 b) 1 c) 3 d) 2 e) 4

Correct Answer: 1

Note: Left can only win by playing in the second column bottom squares.

9. Same game as in the previous question. If the game is played at random, what is the *minimum* number of moves that the game will last?

a) 5 b) 4 c) 3 d) 1 e) 2

Correct Answer: 2

Note: If Left plays in the far right column, then Right plays in the middle row, the game is over.

10. Same game as in the previous question. What is the height of the game tree?

a) 5 b) 6 c) 7 d) 4 e) 8

Correct Answer: 4

Note: 4 moves can be made with one square remaining.

11. Same game as in the previous question. What is the outcome class of this game?

a) $\mathcal{L}$ b) $\mathcal{R}$ c) $\mathcal{N}$ d) $\mathcal{P}$ e) none

Correct Answer: $\mathcal{N}$

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12. Assume that a property X of games is verified to be *false* for games of height 0, 2, 4, 6, 8 and 10. It is then also shown that if property X is assumed to be true for all games of height $n-1$, then property X must be true for all games of height n. Now suppose that G is a particular game of height 12. In addition to the above, what do we need to do in order to prove that G has property X?

a) verify property X for games of height $n = 11$ b) verify property X for games of height $n = 1$ c) verify property X for games of height $n \leq 9$ d) verify property X for games of height $n = 9$ e) nothing

Correct Answer: verify property X for games of height $n = 11$

Note: Only a base case is needed.

13. Let Y be the property that a game is partizan. (A game is partizan if the options are not the same for both players for at least one node of the game tree.) Which of the following fails to be true, or fails to be true by assumption, or fails to be provable, in attempting to prove by induction on game tree height that all games are partizan? Assume that the base case refers to height zero.

i) the base case ii) the induction step iii) the induction hypothesis

a) i) only b) ii) only c) iii) only d) i) and ii) only e) none of the above

Correct Answer: i) only

Note: The zero game is impartial, not partizan. The induction step works, but is useless. The induction step shows that if the options of a game are all partizan then the top node is also partizan, since it only requires that one node in the game tree is partizan.

14. Let Y be the property that a game is in Lwps or Rwps. Which of the following fails to be true, or fails to be true by assumption, or fails to be provable, in attempting to prove by induction on game tree height that all games are in Lwps or Rwps? Assume that the base case refers to height zero.

i) the base case ii) the induction step iii) the induction hypothesis

a) i) only b) ii) only c) iii) only d) i) and ii) only e) none of the above

Correct Answer: ii) only

Note: The base is true since zero is in $\mathcal{P}$. Since the statement is False for all games, it must be that the induction step fails.

15. For a Domineering rectangle of width 1 and height 5, how many Left options are there up to isomorphism? (Equivalently, how many branches to the left of the top node are there in the game tree, after pruning redundant branches?)

a) 0 b) 1 c) 2 d) 3 e) 4

Correct Answer: 1

Note: Every move by Left from the top node will leave a game in which there is one more move for Left to the zero game.

16. Suppose G and H are combinatorial games with G in $\mathcal{L}$ and H in $\mathcal{P}$ or $\mathcal{N}$. What are the *possible* outcomes classes of the game $G + H$?

a) $\mathcal{L}$, $\mathcal{N}$ or $\mathcal{P}$ only b) $\mathcal{L}$ or $\mathcal{P}$ only c) $\mathcal{N}$, $\mathcal{L}$, $\mathcal{R}$ or $\mathcal{P}$ d) $\mathcal{L}$ only e) $\mathcal{N}$ or $\mathcal{L}$ only

Correct Answer: $\mathcal{N}$ or $\mathcal{L}$ only

Note: If H is in P then G+H is in L. If H is in N then G+H is in L or N.

17. Suppose G and H are combinatorial games with G in $\mathcal{N}$ and H in $\mathcal{P}$. What are the *possible* outcomes classes of the game $G + H$?

a) $\mathcal{L}$ or $\mathcal{R}$ only b) $\mathcal{N}$ or $\mathcal{P}$ only c) $\mathcal{N}$, $\mathcal{L}$, $\mathcal{R}$ or $\mathcal{P}$ d) $\mathcal{P}$ only e) $\mathcal{N}$ only

Correct Answer: $\mathcal{N}$ only

18. Suppose H is the 2×7 domineering rectangle. Which of the following columns can Left play on as a winning first move?

i) column 2 ii) column 3 iii) column 4

a) i) and iii) only b) i) only c) ii) and iii) only d) i) and ii) only e) none of them

Correct Answer: i) and iii) only

Note: i) splits the game into two L games, a win for L. ii) splits the game into an N and an R game, a win for R playing first. iii) splits the game into two N games, which L can win playing second by playing in the middle of the summand which R does not play in.

19. Suppose H is the 2×7 domineering rectangle. Suppose that Right's hand is tied by not allowing Right to play moves that cross over columns 4 and 5. Using the One Hand Tied Principle, what conclusion can be made about the 2×7 rectangle?

a) in Rwpf b) in Rwps c) in Lwpf d) in Lwps e) no conclusion

Correct Answer: in Rwpf

Note: R treats the game as a sum of N (2x3) and R (2x4) which they can win playing first. So one-hand-tied says they can also win the 2x7 playing first.

20. Suppose H is the 2×11 domineering rectangle. To show that Lwpf, one can use information about smaller games, plus:

i) playing in the center ii) parity iii) one hand tied

a) i) and iii) only b) i) only c) ii) and iii) only d) i) and ii) only e) iii) only

Correct Answer: i) only

Note: By playing in the center L splits the game into two 2x5 L games, which they can win playing second. It is useless for L to tie their hand or to use parity.